A LIQUEFACTION CRITERION FOR FINE-GRAINED SAND
CONSTITUTING NAM O FORMATION SUBJECTED TO UNI-
DIRECTIONAL AND MULTI-DIRECTIONAL CYCLIC SHEARS

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Abstract. In this paper, fine-grained samples at nominally 50% relative density of Nam O sand were tested using several series of uni-directional and multi-directional cyclic shears. The changes of cyclic shear-induced effective stress reduction were observed for a wide range of shear strain amplitudes and various cyclic shear directions and number of cycles. The effects of such cyclic shearing conditions on the liquefaction resistance of the soil were then clarified. It is indicated from experimental results that the effective stress in Nam O sand reduces quickly by the application of the cyclic shear and that the soil is liquefied even when the cyclic shear strain is at small amplitude (γ = 0.1%). The effects of cyclic shear direction on the effective stress reduction and also on the liquefaction resistance of the soil are evident at small shear strain amplitude; these effects, however, decrease with γ and become negligible when γ ≥ 1.0%, at which the soil is liquefied after a very few numbers of cycles. The occurrence of liquefaction in Nam O sand can be observed precisely for various cyclic shear directions by using relations between the shear strain amplitude and the number of cycles. The liquefaction criterion of Nam O sand was finally obtained and discussed for both cases of uni-directional and multi-directional cyclic shears.

Keywords: cyclic shear, effective stress reduction, liquefaction, relative density, Nam O sand

1 Introduction

Under cyclic loading, pore water pressure might generate and accumulate in saturated sands and clays resulting in the reduction of effective stresses between the soil particles. When the pore water pressure is equal to the total stress, meaning that the effective stress becomes zero, liquefaction is reached and under such a condition, the soil body is under liquid state without any shear strength. For sandy soils with loose density, the reduction of the effective stress may occur suddenly and the liquefaction of the soils is reached easily. The liquefaction of sands and its accompanying hazards have been studied extensively as well as recorded after various dynamic loading events such as earthquakes, pile-driving, ocean waves, etc. [1–3].

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Laboratory experiments are commonly based on the stress-controlled approach, which was usually used to investigate the dynamic properties of clayey soil, e.g. the stress-controlled cyclic simple shear tests by Andersen et al. [4]. As an alternative approach, the strain-controlled tests were introduced by Dobry et al. [5] who confirmed that the shear strain is the main parameter controlling the settlement and pore water pressure generation on the sand during cyclic loading. When focusing on an arbitrary cycle during the cyclic shearing, the deformation of the soil microstructure might increase in proportion to the applied cyclic shear strain amplitude ($\gamma$). From a literature review on the cyclic behaviour of soils, Matsui et al. [6] concluded that the stress-deformation characteristics of clays largely depend on the magnitude of applied shear strain. On the other hand, Matasovic and Vucetic [7] clarified that the generation and the build-up of pore water pressure are a consequence of the tendency of saturated soil to change in volume and that the volume change depends directly on the shear deformation of the soil [8]. These observations mean that the shear strain amplitude is a more fundamental parameter when investigating the pore water pressure during undrained cyclic shear and so, the cyclic strain-controlled tests instead of the stress-controlled ones are the meaningful tool for studying the pore water pressure accumulation under cyclic loading [7].

The dynamic properties of granular materials in general and liquefaction resistance of sand, specifically, have been studied by various testing models. By using the undrained triaxial cyclic shear tests on Toyoura sand, Tatsuoka et al. [9] indicated that the shear strength of the sand increases in proportion to the relative density ($D_r$) and that the cyclic shear resistance of the sand increases quickly when $D_r > 70\%$. By using cyclic simple shear tests in combination with triaxial cyclic shear tests on Fraser Delta sand, Vaid & Sivathayalan [10] showed that the resistance of the sand to cyclic shear increases with $D_r$ for various conditions of lateral stress; meanwhile, for loose density, the influence of lateral stress condition becomes negligible. By using the multi-directional shaking table tests, Pyke et al. [11] concluded that the settlement of sand induced by multi-directional cyclic shear is larger than those generated by uni-directional one. For the irregular cyclic shearing, the maximum shear strain amplitude is considered as the most important parameter governing the settlement of soil deposits [12]. Recently, Matsuda et al. [12, 13] evaluated the effects of the cyclic shear direction and the shear strain amplitude on the dynamic behaviour of artificial and natural sands. The authors indicated that the reduction of the vertical effective stress and the post-cyclic settlement are larger when the soils are subjected to multi-directional cyclic shear. Matsuda et al. [13] investigated the relationships between post-cyclic settlement and effective stress reduction of granular materials by using cumulative shear strain and resultant shear strain, and on the basis of which, an estimation method of effective stress changes was proposed and developed.

In this study, the uni-directional and multi-directional cyclic shear tests were run on a nominally 50% relative density specimen of fine-grained sand for a wide range of shear strain amplitudes ($\gamma$) and the number of cycles ($n$). The effects of such cyclic shearing conditions on the
cyclic resistance (in terms of the vertical effective stress reduction) were then clarified, and on the basis of which, a criterion for the liquefaction resistance was observed and discussed.

### 2 Experimental aspects

The used soil is fine-grained samples of Nam O formation (mQ21 no) (hereinafter called as Nam O sand), which is a kind of marine sand widely distributing along the coastal plains in the Central region of Vietnam (from Quang Ngai province to Quang Tri province). The grain size distribution curve and index properties of the soil, which were determined following Japanese standards [14], are shown in Fig. 1 and Table 1.

![Grain size distribution curve of the testing sample](image)

**Fig. 1.** Grain size distribution curve of the testing sample

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity, $G_s$</td>
<td>2.637</td>
<td>Coefficient of uniformity, $C_u$</td>
<td>2.30</td>
</tr>
<tr>
<td>Maximum void ratio, $e_{\text{max}}$</td>
<td>0.991</td>
<td>Coefficient of curvature, $C_c$</td>
<td>0.91</td>
</tr>
<tr>
<td>Minimum void ratio, $e_{\text{min}}$</td>
<td>0.630</td>
<td>Effective diameter, $D_{10}$ (mm)</td>
<td>0.126</td>
</tr>
</tbody>
</table>

The observations in the field show that the loosest density of the fine-grained sand constituting Nam O formation is about $D_r = 40$–50% (nominally loose density) with natural bulk density of $\rho_t = 1.50$–1.60 g/cm$^3$. Therefore, within the distribution depth as 7.0 meters, which corresponds to the operating depth of the foundation of most structures in these areas, the maximum overburden pressure induced by such saturated soil profile is about $\sigma_{\text{vmax}} = 0.5$ kgf/cm$^2$. Since the relationship between the relative density and liquefaction resistance has been confirmed for granular materials, the above relative density and overburden pressure should be applied for the experiments in this study. In order to prepare testing specimen with pre-
determined volumes of soil, intended to produce the target relative density as $D_r = 45\%$, dried sample of the soil was mixed with de-aired water and kept in a vessel at least for one day before being de-aired in the vacuum cell. The soil sample was then poured into the shear box and pre-consolidated for 15 minutes under the vertical stress as $\sigma_{vo} = 49 \text{kPa} = 0.5 \text{kgf/cm}^2$. After the pre-consolidation with a small amount of vertical settlement, the soil specimen at $D_r = 50 \pm 5\%$ with the dimension of 75 mm in diameter and about 20 mm in height was subjected to the strain-controlled cyclic shear by the multi-directional cyclic shear test apparatus developed at Yamaguchi University, Japan [15]. The membrane-enclosed specimen is surrounded by a stack of acrylic rings (i.e., constant cross-sectional area) and therefore, the volume of the specimen is assumed to be constant by keeping the height of the specimen unchanged during the shear deformation. This condition simulates the undrained cyclic shear. During the constant-volume cyclic shear test, the changes of vertical stress and the pore water pressure at the bottom surface of the specimen were measured.

The experiments were performed under the condition of strain-controlled uni-directional and multi-directional cyclic simple shears. The wave form of the cyclic shear strain was sinusoidal with a period of 2 s. The shear strain amplitude was set in the range from 0.1% to 2.0%, and the number of cycles was fixed in a range from 10 to 150. In the uni-directional cyclic shear tests, the shear strain was applied to the specimen only in one direction and so the orbit of cyclic shear strain forms linear lines. In the multi-directional cyclic shear tests, the shear strain was simultaneously applied to $X$ direction ($\gamma_x$) and $Y$ ($\gamma_y$) direction, which are perpendicular to each other under the same shear strain amplitude but at different phase differences ($\theta$). Then the orbit forms from the linear line for the case of $\theta = 0^\circ$ to elliptical lines when $0^\circ < \theta < 90^\circ$ and circle lines when $\theta = 90^\circ$ which is known as gyratory cyclic shearing. The effects of cyclic shear direction on the dynamic behaviour of sands and clays have been confirmed [12, 13, 15].

### 3 Results and discussions

#### 3.1 Effective stress changes in Nam O sand under uni-directional and multi-directional cyclic shears

Typical records of the effective stress changes in a nominally 50% relative density specimen of Nam O sand subjected to cyclic shear at the shear strain amplitude of $\gamma = 0.1\%$ are shown in Fig. 2a for various cyclic shear directions and the number of cycles. The changes of the vertical effective stress during the early stage of the cyclic shearing are shown more in detail in Fig. 2b.

It is observed in Fig. 2 that the vertical effective stresses generally decrease with the number of strain cycles and that the reductions of the effective stress induced by multi-directional shears are larger than those generated by the uni-directional one. For the multi-directional cyclic shears, the higher degree of phase difference indicates a quicker reduction of
the effective stress. The consequence of these tendencies is that the effective stresses approach zero, and therefore the shear strength of the soil is totally lost and a liquefaction condition is reached.

![Diagram](image1)

**Fig. 2.** Changes of effective stress in the fine-grained specimen at $D_r = 50 \pm 5\%$ of Nam O sand under uni-directional and multi-directional cyclic shears at $\gamma = 0.1\%$

![Diagram](image2)

**Fig. 3.** Effective stress changes versus the number of cycles for the fine-grained specimen at $D_r = 50 \pm 5\%$ of Nam O sand under uni-directional and multi-directional cyclic shears at $\gamma = 0.4\%$ and $1.0\%$

In Fig. 2a, the vertical effective stress gradually decreases with $n$ for various cyclic shear directions when the amplitude of the cyclic shear strain is small ($\gamma = 0.1\%$), and in such cases, the discrepancies in the decreasing tendencies of $\sigma'_v$ and also in the number of cycles required for liquefaction are evident for cyclic shear directions (between uni-direction and multi-direction and between the degrees of phase difference). However, for the larger $\gamma$ such as those in Fig. 3a and 3b ($\gamma = 0.4\%$ and $1.0\%$), the effective stresses suddenly decrease and quickly approach zero,
regardless of the cyclic shear direction. Consequently, the effect of the cyclic shear direction on the effective stress reduction and on the liquefaction resistance of Nam O sand becomes negligible when $\gamma \geq 0.4\%$. These observations indicate that the cyclic shear direction significantly affects the liquefaction resistance of Nam O sand and this influence is dependent on the shear strain amplitude.

3.2 Effective stress reduction ratio versus the number of cycles on Nam O sand subjected to uni-directional and multi-directional cyclic shears

As previously mentioned and explained, the decrement in the effective stress ($|\Delta \sigma'|$) under constant-volume cyclic shearing conditions is assumed to be equal to the increment in the pore water pressure ($U_{\text{dyn}}$) under the undrained conditions. Typical changes of the vertical effective stress reduction ratio, which is defined by $|\Delta \sigma'/\sigma'_{\text{vol}}|$ (being equal to the pore water pressure ratio, defined by $U_{\text{dyn}}/\sigma'_{\text{vol}}$) are shown in Fig. 4 for a nominally 50% density specimen of Nam O sand subjected to the uni-directional and multi-directional cyclic shears with a wide range of shear strain amplitudes and the number of cycles. It is clear that $|\Delta \sigma'/\sigma'_{\text{vol}}|$ increases with the number of cycles, and at the same value of $n$, the larger shear strain amplitudes result in sudden increases of $|\Delta \sigma'/\sigma'_{\text{vol}}|$. In addition, Fig. 4b–4d show that the effective stresses of Nam O sand start to reduce after about 0.1 cycles of the cyclic shear application (equal to 0.2 seconds) and this reduction seems to be independent of the shear strain amplitude and cyclic shear direction. Therefore, $n_{tp} = 0.1$ can be considered as the threshold number of cycles for the effective stress reduction (or the pore water pressure build-up) of Nam O sand under uni-directional and multi-directional cyclic shears.

Liquefaction criterion for the fine-grained specimen at $D_r = 50 \pm 5\%$ of Nam O sand under uni-directional and multi-directional cyclic shears

The relationships between the shear strain amplitude and the number of cycles required for liquefaction of Nam O sand are shown in Fig. 5 for various cyclic shear directions. The symbols in this figure show the observed data and the solid lines corresponding to the average values. It is obvious that the number of cycles required for liquefaction decreases with the shear strain amplitude and that at the same shear strain amplitude, the number of cycles decreases from uni-direction to multi-direction and to the larger phase differences. Therefore, the multi-directional cyclic shear and the phase difference reduce the liquefaction resistance of Nam O sand. Also in Fig. 5, differences of the cycle numbers required for liquefaction between uni-direction and multi-direction ($\Delta n$) are evident at a small shear strain amplitude ($\gamma = 0.1\%$); these differences decrease with $\gamma$ and become negligible when $\gamma \geq 1.0\%$. These differences can be expressed in relation to the shear strain amplitude as $\gamma = 1.4549 \times \Delta n^{-0.704}$ (Fig. 6) or can be eliminated by using the plots in Fig. 7, in which, the shear strain amplitude and the number of cycles are shown in the square-root value for the case of uni-direction.
Fig. 4. Effective stress reduction ratio (or the pore water pressure ratio) versus the number of cycles on Nam O sand subjected to uni-directional and multi-directional cyclic shears.

Fig. 5. Relationship between the number of cycles and shear strain amplitude required for liquefaction in Nam O sand under cyclic shear with various cyclic shear directions.
Fig. 6. $\gamma$ versus $\Delta n$ showing the effect of cyclic shear direction on the liquefaction in Nam O sand

Fig. 7. Relationship between the number of cycles and shear strain amplitude required for liquefaction in Nam O sand after the elimination of the effect of cyclic shear direction

Fig. 8. Relationship between the number of cycles and shear strain amplitude as a liquefaction criterion for fine-grained samples with $D_r = 50 \pm 5\%$ of Nam O sand under uni-directional and multi-directional cyclic shears
Sandy soils are confirmed to be liquefied easily under strong motion, and for fine-grained samples at the nominally 50% density of Nam O sand, liquefaction is initiated after very few numbers of cycles when the shear strain amplitude is larger than 1.0% (Fig. 5), and under such conditions, it becomes difficult to clarify the liquefaction occurrence in the soil. In order to precisely observe the occurrence of liquefaction in Nam O sand for the whole range of shear strain amplitude and the number of cycles, the relationships in Fig. 5 should be plotted in the logarithmic scale such as those in Fig. 8. The relations of $\gamma = 2.9016 \times n^{-0.786}$ and $\gamma = 1.5509 \times n^{-0.771}$ were obtained and considered as the liquefaction criterion of Nam O sand under uni-directional and multi-directional cyclic shears, respectively.

4 Conclusions

In order to clarify the effect of cyclic shear conditions including the cyclic shear direction, shear strain amplitude and the number of cycles on the effective stress changes and on the liquefaction resistance of fine-grained sand at $D_r = 50\% \pm 5\%$ constituting Nam O formation, several series of cyclic simple shear tests were carried out using the multi-directional cyclic simple shear test apparatus. The main conclusions are as follows:

1. The effects of cyclic shear direction on the effective stress reduction and on the liquefaction resistance of fine-grained sand at $D_r = 50\% \pm 5\%$ constituting Nam O formation largely depend on the amplitude of cyclic shear strain. These effects are evident at small shear strain amplitude ($\gamma = 0.1\%$) and decrease and become negligible when $\gamma \geq 1.0\%$.

2. Such effects of the cyclic shear direction on the liquefaction resistance can be eliminated by using the square-root value of $\gamma$ and $n$ for the case of uni-direction.

3. By using the relations of $\gamma$ versus $n$ in the logarithmic scale, the liquefaction condition of the nominally 50% density specimen of fine-grained sand constituting Nam O formation can be observed precisely for the whole range of shear strain amplitude, the number of cycles, and cyclic shear direction.

4. The relations of $\gamma = 2.9016 \times n^{-0.786}$ and $\gamma = 1.5509 \times n^{-0.771}$ are considered as the liquefaction criterion for the nominally 50% density specimen of fine-grained sand constituting Nam O formation subjected to uni-directional and multi-directional cyclic shears, respectively.

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References