

# PAIRING EFFECTS ON NEUTRON ELASTIC SCATTERING AT LOW ENERGIES

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**Abstract.** For the first time, a realistic microscopic calculation for low-energy neutron-nucleus elastic scattering off open-shell nuclei is carried out within the framework of particle-vibration coupling (PVC). In this study, the pairing correlations of the ground state are taken into account. The dependence of the angular distributions on the pairing gaps is discussed.

**Keywords:** elastic scattering, microscopic optical potential, pairing correlations, Skyrme interaction

## 1 Introduction

Nucleon-nucleus ( $NA$ ) scattering is a useful tool to investigate the  $NA$  interaction, as well as the structure of the targets. Most of  $NA$  (in)elastic scattering calculations are based on the optical potential models since they can reduce the complex  $A+1$  problems to 2-body effective problems. At low energies, microscopic optical potentials based on the nuclear structure model are the natural link between nuclear reactions and nuclear structure.

Recently, the energy density functionals built from the nucleon-nucleon ( $NN$ ) phenomenological effective interactions have been successfully applied to  $NA$  scattering off double-closed-shell nuclei. Experimental data have been reproduced with good precision on neutron elastic scattering [1] by  $^{16}\text{O}$ , proton inelastic scattering [2] by  $^{24}\text{O}$ , neutron and proton elastic scattering by

$^{40}\text{Ca}$  and  $^{48}\text{Ca}$  [3, 4, 5], and neutron elastic scattering [6] by  $^{16}\text{O}$  and  $^{208}\text{Pb}$  without ad hoc adjusted parameters. These microscopic-type calculations are quite promising since it opens a possibility to see directly the effects of nuclear structure on the nuclear reaction observables.

Pairing correlation plays a key role to understand the fundamental properties of the structure of nuclei, especially unstable nuclei. So far, this effect has been described by the Hartree–Fock–Bardeen–Cooper–Schrieffer (HF+BCS) [9] approach, the Hartree–Fock–Bogoliubov (HFB) [10] approach, the Highly Truncated Diagonalization Approach (HTDA) [11, 12], and the Quasiparticle Random Phase Approximation (QRPA) [14–16, 19]. Recently, due to pairing, the Fano effects have also been found in the neutron elastic scattering off open-shell nuclei by using the Jost function based on the HFB formalism.

By using the same formalism, Mizuyama et al. [17, 18] show the effect of pairing on the total cross-section and partial cross-section for neutron off open-shell nuclei. They reveal that the resonances can be classified to two types: hole-like and particle-like quasi-particle resonances. Also, the quasiparticles resonances appear as sharp peaks in the total cross-sections of the neutron elastic scattering. However, these calculations still remain a model calculation since they used the Woods–Saxon potential and missed the absorption from the imaginary part of the optical potential.

The goal of the present work is to perform realistic calculations to see the effects of pairing correlations on the angular distributions of elastic scattering. Our models are based on the microscopic optical potential generated from effective Skyrme interactions through the framework of Particle-Vibration Coupling (PVC) on the top of the excited states described by the QRPA calculation. At this step, we take into account the pairing correlations in ground states only.

## 2 Formalism

First, let us briefly recall some general features of the microscopic optical potentials. According to Refs. [6, 7], the MOPs are given as

$$V_{\text{opt}} = V_{\text{HF}} + \Delta\Sigma(\omega), \quad (1)$$

where

$$\Delta\Sigma(\omega) = \Sigma(\omega) - \frac{1}{2}\Sigma^{(2)}(\omega). \quad (2)$$

In Eqs. (1) and (2),  $V_{\text{HF}}$  is the real, local, momentum-dependent, energy-independent Skyrme HF mean-field potential, and  $\omega$  is the nucleon incident energy. The polarization potential,  $\Delta\Sigma(\omega)$ , is non-local, complex, and energy-dependent. The imaginary part of  $\Sigma(\omega)$

is responsible for a loss of the incident flux due to the existence of nonelastic channels.  $\Sigma^{(2)}(\omega)$  is the second-order potential generated from uncorrelated particle-hole contributions.

To take into account the pairing correlations of the targets, the excited states are described by using the QRPA calculations. To do it, we solve the QRPA matrix equation in configuration space. The QRPA operator reads

$$Q^\dagger = \frac{1}{2} \sum_{k,k'} (X_{kk'} \alpha_k^\dagger \alpha_{k'}^\dagger - Y_{kk'}^\dagger \alpha_k \alpha_{k'}), \quad (3)$$

where  $E_n$  is the energy of the  $n$ th phonon state of the target;  $X$ ,  $Y$  are the corresponding forward and backward amplitudes. The QRPA matrix equations have the explicit form

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = E_n \begin{pmatrix} X^n \\ Y^n \end{pmatrix}, \quad (4)$$

$$\begin{aligned} A_{pn,p'n'} &= (E_p + E_n) \delta_{pp'} \delta_{nn'} \\ &+ V_{pp'n'}^J (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) \\ &+ W_{pp'n'}^J (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}), \end{aligned} \quad (5)$$

$$\begin{aligned} B_{pn,p'n'} &= V_{pp'n'}^J (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \\ &+ W_{pp'n'}^J (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}), \end{aligned} \quad (6)$$

where  $p$  and  $p'$  ( $n$  and  $n'$ ) refer to proton (neutron) quasiparticles;  $u$  and  $v$  are the usual BCS occupation factors;  $V^{(J)}$  and  $W^{(J)}$  are coupled p-p and p-h matrix elements, respectively.

The wave function of the incident neutron of mass  $m$ , spin  $\sigma$ , and energy  $\omega$  is

$$\Psi(\cdot, \sigma; \omega) = \sum_{ijm} \frac{u_{ij}(r, \omega)}{r} \mathcal{Y}_{ij}^m(\hat{r}, \sigma), \quad (7)$$

where  $u_{lj}$  is the radial function, and  $\mathcal{Y}$  is the spin-angular part.

We need to solve the Schrödinger equation, which has the form

$$\left[ -\frac{\hbar^2}{2m^*(r)} \nabla^2 - \left( \nabla \frac{\hbar^2}{2m^*(r)} \right) \cdot \nabla + V_{\text{HF}} - \omega \right] \Psi(\sigma; \omega) = -\int \Delta \Sigma(\sigma'; \omega) \Psi(\sigma'; \omega) d^3 r', \quad (8)$$

where  $m^*$  is the effective mass;  $V_{\text{HF}}$  is the Hartree-Fock potential;  $\Delta \Sigma$  is the dynamic part of the microscopic optical potential. This equation is solved by using the DWBA code.

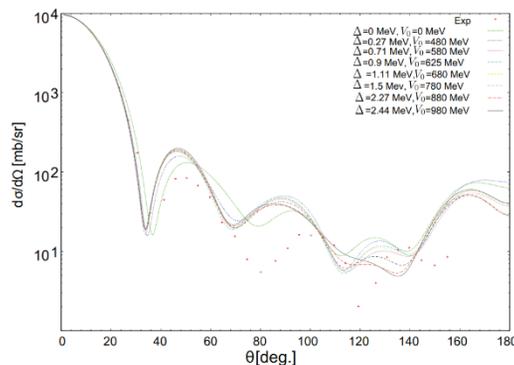
For the numerical calculations, we solve the radial HF equations in the coordinate space: the radial mesh size is 0.1 fm and the maximum value of the radial coordinate is set to be 15 fm. The  $NN$  effective interaction SLy5 has been adopted [8]. After the HF solutions are reached, the ground states and various excited states are then calculated within the fully self-consistent QRPA framework [20]. After obtaining the QRPA excited states, all the natural parity phonons with the multipolarity  $L$  from 0 to 5, whose energies are smaller than 50 MeV, and the fraction of the total isoscalar or isovector strength are larger than 5%, are selected as the inputs for the PVC calculations. The pairing force of surface type reads

$$V = V_0 \left[ 1 - \left( \frac{\rho(\vec{r}_1 + \vec{r}_2)}{\rho_c} \right)^\gamma \right] \delta(\vec{r}_1 - \vec{r}_2) \quad (9)$$

where  $\gamma = 1$ ,  $\rho_c = 0.16 \text{ fm}^{-3}$ ,  $V_0 = 680 \text{ MeV fm}^3$ , which is fitted to reproduce the empirical values of the pairing gaps of  $^{116}\text{Sn}$ .

### 3 Results and discussion

Within the framework of the present study, we focus on the angular distributions, which are the most important nuclear reactions observables. In Fig. 1, we show the angular distributions for the neutron elastic scattering on  $^{116}\text{Sn}$  at several pairing gaps at incident neutron energy 14 MeV. This incident energy is above the energies of the giant resonances. The angular distributions at small scattering angles are better when the pairing is included. There is a systematic disagreement with experimental data at large scattering angles. To understand the effects of pairing in the extreme case, we increase the intensity of the pairing force up to 980 MeV, which corresponds to  $\Delta = 2.44 \text{ MeV}$ . The obtained results show that the pairing has small effects but not negligible on angular distributions. The deviation with the experimental data could be due to some missing structure effects.



**Fig. 1.** Angular distributions of neutron elastic scattering by  $^{116}\text{Sn}$  at different values of pairing gaps. The solid curves show the results of the MOP calculations using the SLy5 interaction. The experimental data are taken from Ref. [21]

To see the effects of pairing on the absorption part of the microscopic optical potential, we define the quantity

$$W(R, s) = \sum_{ij} \frac{2j+1}{4\pi} \text{Im}\Delta\Sigma_{ij}(r, r', \omega), \quad (10)$$

where  $R = \frac{1}{2}(r + r')$  corresponds to the radius and shape of  $\text{Im}\Delta\Sigma$ , and  $s = r - r'$  shows its non-locality. In Fig. 2, we show the quantity  $W(R, s = 0)$  at different values of the pairing gaps. It is very interesting to see that the pairing

increases the absorption on the surface while it reduces the absorption interior.

This work opens a possibility to understand the reactions observables from the nuclear structure view. As the first step, we only take into account the pairing correlations of the ground states. Hopefully, we could extend the work of Mizuyama et al. [17, 18] (T. V. Nhan Hao is one of the authors of this work) by extending the Jost function framework based on the PVC (including the pairing) and the effective Skyrme interaction to study the pairing effects in a more complete model.

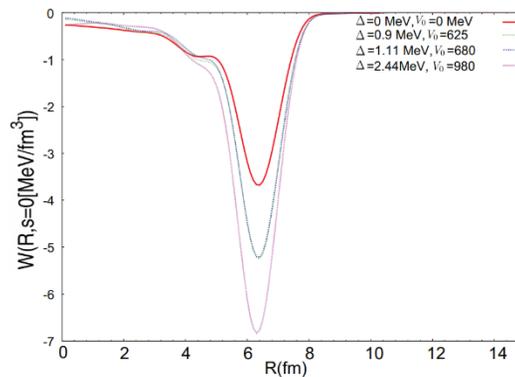


Fig. 2. The calculated  $W(R, s = 0)$  by  $^{116}\text{Sn}$  at different values of pairing gaps at neutron incident energy 14 MeV. The interaction SLy5 has been used.

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