

# STRUCTURE, STABILITY, AND ELECTRONIC PROPERTIES OF SINGLY AND DOUBLY TRANSITION-METAL-DOPED BORON CLUSTERS $B_{14}M$

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**Abstract.** An examination of the first-row-transition-metal-doped boron clusters,  $B_{14}M$  ( $M = \text{Sc, Ti, V, Cr, Mn, Fe, Co, Ni, and Cu}$ ) in the neutral state, is carried out using DFT quantum chemical calculations. The lowest-energy equilibrium structures of the clusters considered are identified at the TPSSh/ 6-311+G(d) level. The structural patterns of doped species evolve from exohedrally capped quasi-planar structure  $B_{14}$  to endohedrally doped double-ring tubular when  $M$  is from Sc to Cu. The  $B_{14}\text{Ti}$  and  $B_{14}\text{Fe}$  appear as outstanding species due to their enhanced thermodynamic stabilities with larger average binding energies. Their electronic properties can be understood in terms of the density of state.

**Keywords:** DFT, boron cluster, density of state

## 1 Introduction

There has been considerable interest in the boron-based clusters as endorsed by a large number of experimental and theoretical investigations in the last decades. This is due to not only their novel physical and chemical properties but also their promising abilities for new technological applications. The structural landscape of small pure boron clusters up to  $B_{20}$ , provided by many studies [1], is now clearly determined for both neutral and charged states. It reveals that from the size  $B_{17}^+$  to  $B_{20}^+$ , the cations favor a double ring tubular structure [2], whereas anionic and neutral clusters are more stable in the planar form [3, 4] except for the neutral  $B_{14}$ . The  $B_{14}$  is an extraordinary size during the growth mechanism of small bare boron clusters since it is the smallest all-boron fullerene [5], whereas the dicationic state

$B_{14}^{2+}$  was found as the first double-ring (DR) boron cluster [6]. The DR structure emerges from a superposition of two  $B_k$  strings leading to a tube  $B_{2k}$ . The most stable structure of neutral  $B_{20}$ , having the very high stability in comparison with the other isomers, is the most well-known all-boron double ring [7], among the others  $B_{18}^{2+}$  [6],  $B_{22}^{2+}$  [8],  $B_{24}$  [9], etc. For the neutral state of pure boron clusters  $B_n$ , however, the DR structures only exist at the sizes  $n \geq 20$ . The DR tube achieves double aromaticity [10–12] by the classic Hückel ( $4N + 2$ ) rule for both  $\pi$  electrons (radial electrons) and  $\sigma$  electrons (tangential electrons). It can be thus rationalized for the enhanced stability of the DR structure.

The first-row transition metals, including Sc, Ti, V, Cr, Mn, Fe, Co, Ni, and Cu, which have the unpaired valence electrons  $4s^23d^1$ ,  $4s^23d^2$ ,  $4s^23d^3$ ,  $4s^13d^5$ ,  $4s^23d^5$ ,  $4s^23d^6$ ,  $4s^23d^7$ ,  $4s^23d^8$ , and  $4s^13d^{10}$ , respectively, are interesting magnetic elements.

They are expected to become the potential candidates as dopants in clusters due to the interaction between these impurities and host electrons and may alter both electronic and geometrical structures and thus generate the doped cluster possessing the novel physico-chemical properties [13, 14].

Numerous theoretical and experimental studies reported that doping one transition-metal atom on small boron clusters leads to the formation of the wheel-type structures, detected at the sizes of  $8 \leq n \leq 10$ , in which the impurity M tends to be encapsulated at the center of the  $B_n$  rings [15–20]. For the sizes of  $B_n$  with  $n > 10$ , numerous geometrical patterns of boron clusters doped with a transition metal were found, such as the leaf-like, pyramid-like, umbrella-like, or metallo-borophene structures [21–23]. Remarkably, our previous study indicates that the iron-doped  $B_{14}Fe$  and  $B_{16}Fe$  are stabilized DR tubes, whereas  $B_{18}Fe$  and  $B_{20}Fe$  are stabilized fullerenes [24]. Most recently, our systematic investigation on singly and doubly nickel-doped boron clusters reveals that from the size  $n = 14$ , the Ni impurities cause stronger effects, and the most stable isomers  $B_nNi_m$  thus favor the shape of the related DR tubular boron structures [25]. The formation and high thermodynamic stability of boron clusters doped with both Fe and Ni certify the use of transition-metal atoms as impurities to generate various growth paths leading to larger boron clusters possessing peculiar 3D structures, such as tubes, cages, or fullerenes [26].

Although some studies on transition-metal-doped boron clusters have been carried out, the investigations on metal-doped boron clusters, in particular at the sizes  $n > 10$ , are insufficient. There are still some boron clusters doped with  $3d$  transition metals that have not been systematically examined yet. Only a few  $B_nM_m$  clusters, with M being a transition metal, such as Sc, Ti, Fe, Co, and

Ni, were reported in the recent past [25, 27, 28]. Motivated by that, we set out to operate a theoretical study on the boron clusters doped with a transition metal atom  $B_{14}M$ , where M is a first-row transition metal ranging from Sc to Cu, using density functional theory (DFT) calculations. We thoroughly identify the geometries of the most stable structures and, thereby, explore their exciting possibilities of structural evolution as well as determine their electronic configuration and energetic parameters.

## 2 Computational Methods

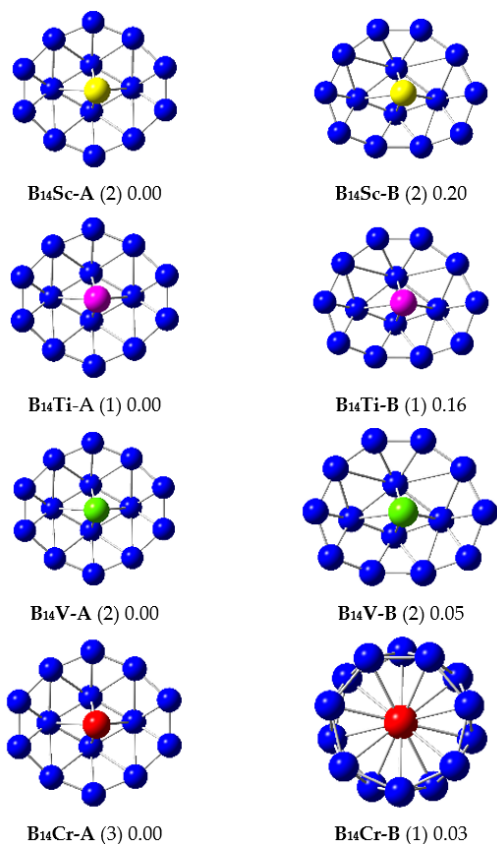
In consideration of the reliability tests obtained from many earlier reports on boron-based clusters [8, 24, 25, 27–29], we have used the hybrid TPSSh functional in conjunction with the 6-311+G(d) basis sets as implemented in the Gaussian 09 package [30] for all calculations in this work. The search for energy minima is conducted using two diverse approaches. First, all possible structures of  $B_nM_m$  clusters are generated using a stochastic algorithm [31]. In addition, initial structures of  $B_nM_m$  are manually composed by adding M-atoms at all possible positions on the surfaces of the known  $B_{14}$  structures. The harmonic vibrational frequencies of  $B_nM_m$  are afterward identified at the same level.

For the analysis of the electronic distribution, we use the electronic density of state (DOS) approach. The values of DOSs are also obtained using TPSSh/6-311+G(d) computations.

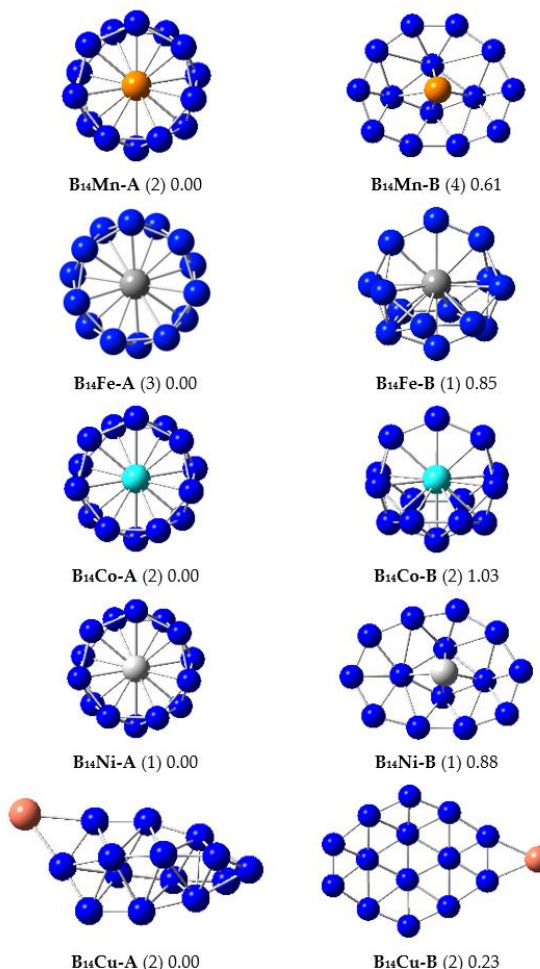
## 3 Results and discussion

### 3.1 Lower-lying isomers of $B_{14}M$ clusters

The shapes of the equilibrium structures of the  $B_{14}M$  clusters detected, their spin states, and DFT relative energies are shown in **Fig. 1** and **Fig. 2**. Because of a large number of isomers located on the potential energy surfaces of the clusters



**Fig. 1.** Shapes, spin states (in the brackets), and relative energies ( $\Delta E$ , eV) of the lower-lying isomers  $B_{14}M$  with  $M = \text{Sc, Ti, V, and Cr}$ .  $\Delta E$  values are obtained from TPSSh/6-311+g(d) + ZPE computations



**Fig. 2.** Shapes, spin states (in the brackets), and relative energies ( $\Delta E$ , eV) of the lower-lying isomers  $B_{14}M$  with  $M = \text{Mn, Fe, Co, Ni, and Cu}$ .  $\Delta E$  values are obtained from TPSSh/6-311+g(d) + ZPE computations

considered, only the ground state and the second lower-lying isomer whose relative energy is closest to the corresponding ground state isomer are presented for each size. Conventionally, a  $B_{14}M-X$  label is used for each isomer of the  $B_{14}M$  clusters considered, where  $M$  is **Sc, Ti, V, Cr, Mn, Fe, Co, Ni, and Cu**, and  $X = A$  and  $B$  referring to the different isomers with increasing relative energy. The main geometrical characteristics can briefly be described as follows:

As  $M$  is, in turn, Sc, Ti, V, and Cr, the most stable isomers of  $B_{14}M$  prefer the structure in which the dopant  $M$  is capped on the surface of the quasi-planar shape of cation  $B_{14}^+$  [2]. DFT calculations

indicate two degeneracies in energy for  $B_{14}V$  and  $B_{14}Cr$ . Interestingly, while both  $B_{14}V-A$  and  $B_{14}V-B$ , being energetic degenerated with a gap of 0.05 eV, still have the quasi-planar shapes of the  $B_{14}$  framework, there is a structure competition at  $B_{14}Cr$ . Accordingly, the triplet spin state  $B_{14}Cr-A$  continues the quasi-planar  $B_{14}$  skeleton like the  $B_{14}M$  described above, whereas the closed-shell spin state  $B_{14}Cr-B$ , being only 0.03 eV higher in energy than  $B_{14}Cr-A$ , possesses a DR structure composed of two seven-membered rings in an anti-prism disposition [6] and a Cr atom is encapsulated at the center of the tubular.

Similar to **B<sub>14</sub>Cr-B**, the lowest-lying isomers of next B<sub>14</sub>M clusters with M being Mn, Fe, Co, and Ni are also generated by putting the dopant M in the center of the DR cylinder B<sub>14</sub>. Among them, the triplet spin state **B<sub>14</sub>Fe-A** and the closed-shell electronic configuration **B<sub>14</sub>Ni-A** are reported in our previous studies [24, 25]. The remaining isomers with different geometrical structures are much less stable with a large energy gap, being at least 0.61 eV.

In the family B<sub>14</sub>M with M ranging from Sc to Cu, only the most stable structure of B<sub>14</sub>Cu keeps the fullerene-like geometry of pure neutral B<sub>14</sub> [5]. The isomer **B<sub>14</sub>Cu-A**, formed by adding a Cu atom on an edge of the fullerene framework B<sub>14</sub>, is 0.23 eV lower in energy than **B<sub>14</sub>Cu-B**, also formed by adding a Cu atom on an edge of the quasi-planar structure of B<sub>14</sub><sup>+</sup> [2].

Generally, the doping of B<sub>14</sub> successively with different first-row transition metals ranging from Sc to Cu tends to make the DR structure, in which the metal dopant is located at the center of DR B<sub>14</sub> tubular. For three lightest dopants, including Sc, Ti, and V, the DR shape has not appeared yet. For M = Cr, however, there is a structure competition because the DR structure is almost as stable as the quasi-planar structure. Subsequently, the calculated results of B<sub>14</sub>M with M being, in turn, Mn, Fe, Co, and Ni, show the strong domination of DR structure. It can be understood by the fact that the atomic radius of Sc, Ti, and V is longer than that of the remaining 3d metals. Hence, the hollow volume inside the B<sub>14</sub> DR is not large enough to confine these metal impurities, whereas the heavier dopants (M = Cr, Mn, Fe, Co, and Ni) with shorter atomic radius can be encapsulated at the center of DR B<sub>14</sub>. Moreover, the B<sub>14</sub>Cr can be considered as a “critical point” of the B<sub>14</sub>M series since both DR and quasi-planar shapes exist together. The B<sub>14</sub>Cu species is an exception because its geometrical structure is

different from DR. It can be rationalized by the fact that the copper atom with 4s<sup>1</sup>3d<sup>10</sup> electronic configuration can easily lose one valence electron to get the full-filled configuration and behaves as an electron donor. Therefore, the Cu atom favors adsorption on a bridge site of the fullerene B<sub>14</sub> framework.

### 3.2 Relative stabilities of B<sub>14</sub>M

Like in previous studies on various clusters [25, 32, 33], the relative stabilities of B<sub>14</sub>M species considered can be evaluated on the basis of the average binding energy per atom ( $E_b$ ), which is conventionally defined as follows:

$$E_b(\text{B}_{14}\text{M}) = [14E(\text{B}) + E(\text{M}) - E(\text{B}_{14}\text{M})]/15 \quad (1)$$

Furthermore, the average binding energy of pure boron neutral B<sub>15</sub> with the same number of atoms is also determined for comparison with  $E_b(\text{B}_{14}\text{M})$ :

$$E_b(\text{B}_{15}) = [15E(\text{B}) - E(\text{B}_{15})]/15 \quad (2)$$

where  $E(\text{B})$  and  $E(\text{M})$  are the total energy of the B-atom and M-atom, respectively.  $E(\text{B}_{14}\text{M})$  and  $E(\text{B}_{15})$  are the total energy of the neutral B<sub>14</sub>M and B<sub>15</sub>, respectively. All these energetic values are obtained from TPSSH/6-311+G(d) + ZPE calculations, and the values of  $E_b(\text{B}_{14}\text{M})$  with M being from Sc to Cu, in comparison with  $E_b(\text{B}_{15})$ , are illustrated in Fig. 3. The coordinate of the geometry of neutral B<sub>15</sub> is taken from a previous study of Tai et al. [4]

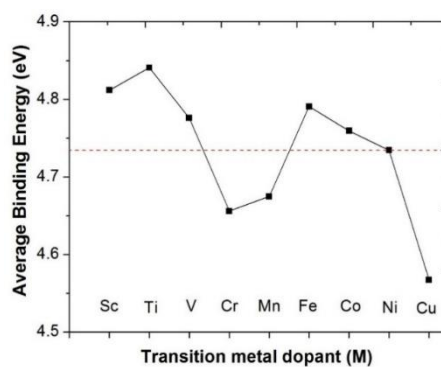


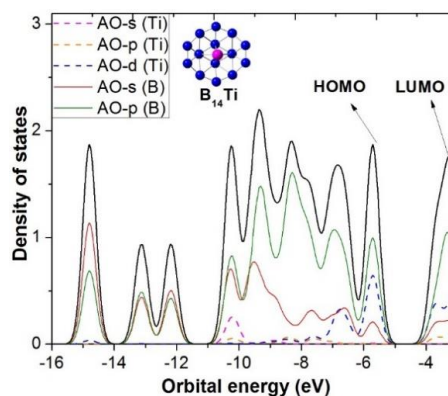
Fig. 3. Average binding energies ( $E_b$ , eV) of 3d transition metal doped B<sub>14</sub>M

Fig. 3 shows except for the  $E_b$  values of  $B_{14}Ni$  and  $B_{15}$  being almost equal, the  $E_b$  values of  $B_{14}Sc$ ,  $B_{14}Ti$ ,  $B_{14}V$ ,  $B_{14}Fe$ , and  $B_{14}Co$  species are higher than those of  $B_{15}$ , whereas the  $E_b$  values of  $B_{14}Cr$ ,  $B_{14}Mn$ , and  $B_{14}Cu$  are lower than those of  $E_b(B_{15})$ . In other words, while Sc, Ti, V, Fe, and Co dopants increase the cluster stability concerning fragmentations, Cr, Mn, and Cu tend to decrease it. In addition, when M goes successively from Sc to Cu, the  $E_b(B_{14}M)$  gets the maximum value of cluster stability at  $B_{14}Ti$ . It decreases from  $B_{14}Ti$  to  $B_{14}Cr$  and then increases again from  $B_{14}Cr$  to  $B_{14}Fe$ . From Fe to Cu, the cluster stability of  $E_b(B_{14}M)$  continuously decreases. In particular, it strongly decreases from  $B_{14}Ni$  to  $B_{14}Cu$  and gets the smallest value at  $B_{14}Cu$ . This proves that Cu dopant prefers to donate electrons instead of making chemical bonds.

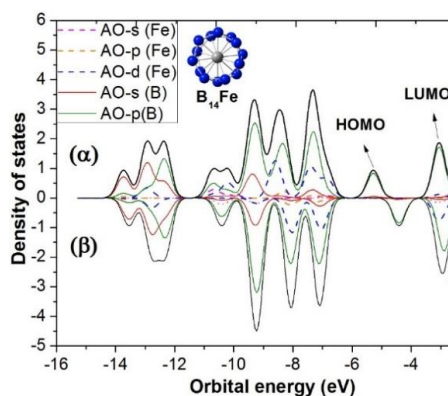
### 3.3 Density of states of $B_{14}Ti$ and $B_{14}Fe$

The picture of the binding energy of  $B_{14}M$  reveals that both  $B_{14}Ti$  clusters – closed-shell electronic configuration and the high spin state  $B_{14}Fe$  – exhibit the enhanced thermodynamic stability with higher average binding energies. They have typical geometric structures, in which the Ti dopant is capped on the surface of the quasi-planar  $B_{14}$ , whereas the Fe dopant is located at the center of a  $B_{14}$  DR. To achieve more insights into the relative stability of the clusters considered, we now examine their molecular orbital pictures under the viewpoints of the jellium shell model [34], in which the total density of states of a molecular system can be considered as an energy spectrum of its molecular orbitals (MOs), whereas the partial density of states (pDOS) is figured out only from relevant atomic orbitals and thereby shows the composition of the MOs involved.

Fig. 4 shows both partial and total densities of states of the singlet  $B_{14}Ti$ -A and the triplet DR  $B_{14}Fe$ -A, in which the  $\alpha$  and  $\beta$  spin MOs are separately plotted. This interprets a clear picture of



a) Total (DOS) and partial (pDOS) of  $B_{14}Ti$



b) Total (DOS) and partial (pDOS) of  $B_{14}Fe$

Fig. 4. Total (DOS) and partial (pDOS) densities of state of (a)  $B_{14}Ti$  and (b)  $B_{14}Fe$

their electronic shells. As expected, the frontier MOs are composed mainly of the 2p(B) and 3d AOs of Ti or Fe dopant but with a larger component of the boron AOs. The HOMO and LUMO of  $B_{14}Fe$ , however, appear particularly from the boron AOs, whereas the HOMO and LUMO of  $B_{14}Ti$  are composed predominantly of 2p(B), 3d(Ti), and, to a lesser extent, of 2s(B) AOs.

## 4 Concluding Remarks

In this investigation, both geometrical and electronic structures of the first-row-transition-metal-doped boron  $B_{14}M$  clusters, where M is, in turn, Sc, Ti, V, Cr, Mn, Fe, Co, Ni, and Cu in the neutral state, were examined using the quantum

chemical DFT approach. The clusters with lighter dopants (M = Sc, Ti, and V) prefer the capped quasi-planar structure, while the heavier ones (M = Cr, Mn, Fe, Co, and Ni) favor a DR structure, in which the metal dopant is located at the center of the DR B<sub>14</sub> tubular. The cluster B<sub>14</sub>Cr can be considered as a critical point due to the structure competition between endohedrally doped DR and exohedrally capped quasi-planar structure. The B<sub>14</sub>Cu, which is formed by adding the Cu dopant on an edge of the fullerene B<sub>14</sub>, is an exception. The results also indicate that besides the typical geometric features, both B<sub>14</sub>Ti and B<sub>14</sub>Fe species have enhanced thermodynamic stabilities with high average binding energies. Their MO properties, thus, are examined from the density of state approaches.

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