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HIGHER-ORDER NONCLASSICAL AND ENTANGLEMENT PROPERTIES IN PHOTON-ADDED TRIO COHERENT STATE

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Abstract. This paper studies the higher-order nonclassical and entanglement properties in the photon-added trio coherent state (PATCS). We use the criterion of higher-order single-mode antibunching to evaluate the role of the photon addition operation. Furthermore, the general criteria for detection of higher-order three-mode sum squeezing and entanglement features in the PATCS are also investigated. The results show that the photon addition operation to a trio coherent state can enhance the degree of both the higher-order single-mode antibunching and the higher-order three-mode sum squeezing and enlarge the value of the higher-order three-mode entanglement factor in the photon-added trio coherent state. In addition, the manifestation of the single-mode antibunching and the entanglement properties are more obvious with increasing the higher values of orders.

Keywords: Photon-added trio coherent state, higher-order nonclassical properties, antibunching, sum squeezing, entanglement

1 Introduction

The nonclassical and entanglement properties of the nonclassical states have been applied to the quantum tasks in the quantum optics and quantum information, such as using antibunching for the generation of the single-photon sources [1], squeezing for the detection of the gravitational waves in the LIGO interferometer [2], and exploiting the entanglement for the implementation of protocols in quantum teleportation [3] and quantum secret sharing [4]. Therefore, the study of nonclassicality and entanglement of nonclassical states is an important work in the discovery of the quantum optics. It has been known that a classical state (e.g., coherent or thermal state) is transformed into a nonclassical one by adding photons on it [5, 6]. As a further development, the addition of photons on two-mode states was studied and investigated, such as the photon-added pair coherent states [7], the photon-added displaced squeezed states [8], and the photon-added squeezed vacuum state [9]. Thanks to the photon addition operation, quantum features in these states, for example, the degree of the squeezing and the entanglement behaviours were enhanced [8, 9]. This is meaningful in the processes of quantum information and computation, e.g., improving the quantum key distribution protocol [10]. Keeping this in mind, we study the addition of photon to three-mode states. Obviously, the three-mode states play a central role in the network tasks of quantum information, including controlled teleportation [11] and joint remote state preparation [12]. Therefore, enhancing the nonclassical and entanglement properties of these states will raise the effectiveness of the applications. In the class of three-mode non-Gaussian states, the trio coherent state (TCS) is an important state of the boson field [13]. The TCS is given in terms of Fock states as follows:

$$|\psi_{p,q}\rangle_{abc} = \sum_{n=0}^{\infty} c_n \, |n, n+p, n+p+q\rangle_{abc}, \quad (1)$$

with p and q being non-negative integers, and the coefficient c_n is given in the term

$$c_n = \frac{N_{p,q}\xi^n}{\sqrt{n!(n+p)!(n+p+q)!}}, \quad (2)$$

where $\xi = re^{i\varphi}$ with r and φ being real, $|n, n+p, n+p+q\rangle = |n\rangle|n+p\rangle|n+p+q\rangle$ is denoted as the three-mode Fock state, and $N_{p,q}$ is the normalized factor of the TCS given by

$$N_{p,q}^{-2} = \sum_{n=0}^{\infty} \frac{r^{2n}}{n!(n+p)!(n+p+q)!}. \quad (3)$$

The TCS is defined as the right eigenstate simultaneously of operators $\hat{a}\hat{b}\hat{c}$, $\hat{N}_b - \hat{N}_a$, and $\hat{N}_c - \hat{N}_b$, corresponding to eigenvalues ξ, p, and q, respectively, i.e., satisfying equations $\hat{a}\hat{b}\hat{c}\,|\Psi_{p,q}\rangle_{abc} = \xi\,|\Psi_{p,q}\rangle_{abc}$,

$(\hat{N}_b - \hat{N}_a)\,|\Psi_{p,q}\rangle_{abc} = p\,|\Psi_{p,q}\rangle_{abc}$,

and $(\hat{N}_c - \hat{N}_b)\,|\Psi_{p,q}\rangle_{abc} = q\,|\Psi_{p,q}\rangle_{abc}$,

in which $\hat{N}_x = \hat{x}^+\hat{x}$, $\hat{x}^+$ $(\hat{x})$ is the bosonic creation (annihilation) operator of mode x, $x = \{a, b, c\}$. Some nonclassical properties of the TCS in both usual and higher orders were investigated in [13, 14]. Therein, the single-mode squeezing, the two-mode squeezing, as well as the three-mode sum squeezing do not exist in such a state. Besides, an experimental scheme for the generation of the TCS has been introduced [15]. Therefore, the addition of photons to the TCS may be feasible by using the protocol of Zavatta et al. [6].

Recently, a photon-added trio coherent state (PATCS) has been introduced [16]. The PATCS is written as follows:

$$|\psi_{p,q}; h,k,l\rangle_{abc} = N_{p,q;h,k,l}\,\hat{a}^{+h}\hat{b}^{+k}\hat{c}^{+l}\,|\psi_{p,q}\rangle_{abc}, \quad (4)$$

where $N_{p,q;h,k,l}$ is the normalized factor; h, k, and l are non-negative integers, which are referred to the number of photons added. In terms of the Fock states, the PATCS is given by

$$|\psi_{p,q;h,k,l}\rangle_{abc} = \sum_{n=0}^{\infty} c_{n;h,k,l}\,|n+h, n+p+k, n+p+q+l\rangle_{abc}, \quad (5)$$

with $c_{n;h,k,l}$ being determined as follows:

$$c_{n;h,k,l} = N_{p,q;h,k,l}\,c_n \frac{\sqrt{(n+h)!(n+p+k)!(n+p+q+l)!}}{\sqrt{n!(n+p)!(n+p+q)!}}. \quad (6)$$

The normalized condition leads to $\sum_n c^2_{n;h,k,l} = 1$, thus

$$N_{p,q;h,k,l}^{-2} = \sum_{n=0}^{\infty} \frac{|c_n|^2 (n+h)!(n+p+k)!(n+p+q+l)!}{n!(n+p)!(n+p+q)!}. \quad (7)$$

It is easy to know that the PATCS is reduced to the TCS if h = k = l = 0. In the PATCS, the quantum average of operators $\hat{a}^{+i_a}\hat{a}^{i_a}\hat{b}^{+i_b}\hat{b}^{i_b}\hat{c}^{+i_c}\hat{c}^{i_c}$ is calculated as follows:

$$B_{i_a,i_b,i_c} = \langle \hat{a}^{+i_a}\hat{a}^{i_a}\hat{b}^{+i_b}\hat{b}^{i_b}\hat{c}^{+i_c}\hat{c}^{i_c}\rangle = \sum_{n=X}^{\infty} \frac{|c_{n;h,k,l}|^2 (n+h)!(n+p+k)!(n+p+q+l)!}{(n+h-i_a)!(n+p+k-i_b)!(n+p+q+l-i_c)!}, \quad (8)$$

where i_a, i_b, and i_c are non-negative integers, and $X = \max(0, i_a - h, i_b - k - p, i_c - p - q - l)$.

Some usual nonclassical and entanglement properties in the PATCS, such as the Wigner distribution function, the three-mode sum squeezing, and the three-mode entanglement, have been studied in detail in [16]. In this paper, we focus on the study of the higher-order nonclassical,

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as well as entanglement properties in the PATCS. We investigate the higher-order single-mode antibunching property in Section 2. Section 3 presents the higher-order three-mode sum squeezing behaviours. Section 4 clarifies the higher-order entanglement characteristic. Finally, we briefly summarize the main results of the paper in the conclusions.

2 Higher-order single-mode antibunching

Antibunching property plays an important role in the quantum processes. For example, it was exploited to generate the photon-added states via the beamsplitters [17]. The criterion for the detection of antibunching was first introduced by Lee [18], then further extended by others [14]. According to An [14], the factor to determine the antibunching degree of mode x in higher-order i is given by

$$A_{x;i} = \frac{\langle \hat{N}_x^{(i+1)} \rangle}{\langle \hat{N}_x^{(i)} \rangle \langle \hat{N}_x \rangle} - 1, \tag{9}$$

where $\langle \hat{N}_x^{(i)} \rangle = \langle \hat{N}_x(\hat{N}_x - 1)\ldots(\hat{N}_x + 1 - l) \rangle = \langle \hat{a}_x^{+i} \hat{a}_x^{i} \rangle$, $\langle \cdots \rangle$ denotes the quantum average. A certain state exists with the higher-order single-mode antibunching (HOSMA) when $A_{x;i} < 0$; the more negative the $A_{x;i}$ is, the larger the degree of HOSMA will be. Let us consider the PATCS, from Eq. (8), the factor measuring the degree of HOSMA in mode a is given as

$$A_{a;i} = \frac{B_{i+1,0,0}}{B_{i,0,0} B_{1,0,0}} - 1. \tag{10}$$

Similarly, with respect to mode b, we obtain

$$A_{b;i} = \frac{B_{0,i+1,0}}{B_{0,i,0} B_{0,1,0}} - 1. \tag{11}$$

For mode c, it is determined as

$$A_{c;i} = \frac{B_{0,0,i+1}}{B_{0,0,i} B_{0,0,1}} - 1. \tag{12}$$

We use the analytical expressions in Eqs. (10)–(12) to investigate the property of the HOSMA in the PATCS. For mode a, Figure 1 plots the dependence of factor $A_{a;i}$ on r with $p = q = 0$ for several values of h, k, and l, in which the case $h = k = l = 0$ corresponds to the TCS, while others are the PATCS.

There are some prominent points in the property of the HOSMA in the PATCS. Firstly, antibunching is found in any higher-orders. When i becomes bigger, factor $A_{a;i}$ is more negative. However, the degree of HOSMA is reduced by increasing r. Secondly, in the small region of r, the photon addition operation can enhance the degree of HOSMA. On the other hand, the bigger the photon number added, the more negative the factor $A_{a;i}$ will become. Nevertheless, in the large area of r, the addition of photons reduces the degree of HOSMA. Finally, the degree of HOSMA also depends on the way of photon-adding. As shown in Figure 2, we plot factor $A_{a;i}$ as a function of r with $p = q = 0$ and $i = 3$ for fixed $h + k + l = 6$. It is worth noting that the above discussions remain true for mode b and c.

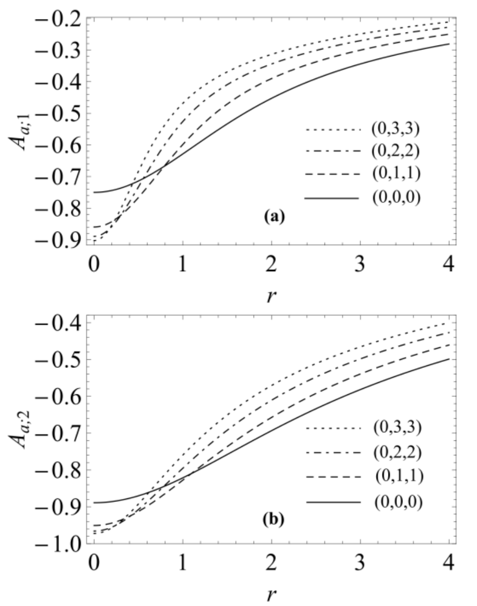

Fig. 1. Factor $A_{a;i}$ as function of r with $p = q = 0$ and in (a) $i = 1$, and in (b) $i = 2$ for $(h,k,l) = (0,0,0)$ (the solid line), (0,1,1) (the dashed curve), (0,2,2) (the dot-dashed curve), and (0,3,3) (the dotted curve)

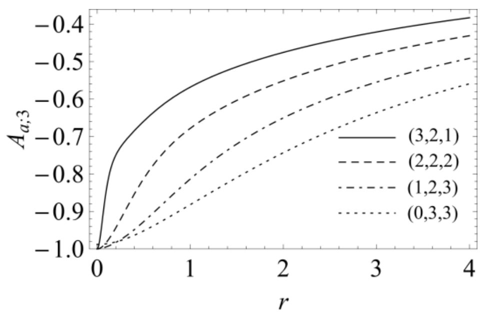

Fig. 2. Factor $A_{a;i}$ as function of r with $p = q = 0$ and $i = 3$ for $(h,k,l) = (3,2,1)$ (the solid line), (2,2,2) (the dashed curve), (1,2,3) (the dot-dashed curve), and (0,3,3) (the dotted curve)

3 Higher-order three-mode sum squeezing

Squeezing property was applied in numerous quantum tasks [19]. Various criteria for the detection of squeezing were introduced and investigated, such as sum squeezing, difference squeezing, single-mode squeezing, multimode squeezing, usual squeezing, and higher-order squeezing [8, 14]. In this section, we define a generalized criterion for the detection of higher-order three-mode sum squeezing. Let us consider two orthogonal Hermitian operators

$$\hat{X} = \frac{\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c} + \hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}}{2}, \quad \hat{Y} = \frac{i(\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c} - \hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c})}{2}, \tag{13}$$

where j_a, j_b, and j_c are non-negative integers. The above operators obey the commutative relation

$$[\hat{X},\hat{Y}] = \frac{i}{2}\hat{Z}, \tag{14}$$

in which

$$\hat{Z} = \hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c} - \hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}. \tag{15}$$

A state exists with higher-order three-mode sum squeezing in $\hat{X}$ or $\hat{Y}$ if it satisfies

$$S_X = \frac{4\langle(\Delta\hat{X})^2\rangle - \langle\hat{Z}\rangle}{|\langle\hat{Z}\rangle|} < 0,$$

or

$$S_Y = \frac{4\langle(\Delta\hat{Y})^2\rangle - \langle\hat{Z}\rangle}{|\langle\hat{Z}\rangle|} < 0. \tag{16}$$

Factor S_X or S_Y also manifests the degree of higher-order three-mode sum squeezing. The more negative these factors are, the higher the squeezing degree will become. Note that in case $j_b = j_c = 0$ or $j_a = j_b = j_c$, the above criteria correspond to the higher-order single-mode squeezing or the higher-order three-mode sum squeezing (HOTMSS) [14]. However, when $j_a = j_b$ and $j_c = 0$, they become the higher-order two-mode sum squeezing criteria [8]. In the PATCS, the inequalities in Eq. (16) are written as

$$S_X = \frac{2\langle(\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c})^2\rangle - 4\langle\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\rangle^2 + 2\langle\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}\rangle}{|\langle\hat{Z}\rangle|} < 0, \tag{17}$$

or

$$S_Y = \frac{-2\langle(\hat{a}^{+x}\hat{b}^{+y}\hat{c}^{+z})^2\rangle + 2\langle\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}\rangle}{|\langle\hat{Z}\rangle|} < 0. \tag{18}$$

In the PATCS, because

$\langle(\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c})^2\rangle = \langle\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\rangle = 0$ if $j_a \neq j_b \neq j_c$, and the value of the quantum mean of operators $\hat{a}^{+j_a}\hat{b}^{+j_b}\hat{c}^{+j_c}\hat{a}^{j_a}\hat{b}^{j_b}\hat{c}^{j_c}$ is non-negative. Therefore, in this case, the PATCS does not exist with HOTMSS in both $\hat{X}$ and $\hat{Y}$. However, if $j_a = j_b = j_c = j > 0$, from the analytical expression in Eq. (8), the factors of the HOTMSS in the PATCS are given by

$$S_{X;j} = \frac{2D_{0,2j} - 4D_{0,j}^2 + 2B_{j,j,j}}{|D_{j,j} - B_{j,j,j}|}, \tag{19}$$

and

$$S_{Y;j} = \frac{-2D_{0,2j} + 2B_{j,j,j}}{|D_{j,j} - B_{j,j,j}|}, \tag{20}$$

with

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$$\begin{aligned}D_{i_1,i_2} &= \langle(\hat{a}\hat{b}\hat{c})^{i_1}(\hat{a}^{+}\hat{b}^{+}\hat{c}^{+})^{i_2}\rangle \\ &= \sum_{n=\max(0,i_1-i_2)}^{\infty} c_{n;h,k,l}c_{n+i_2-i_1;h,k,l}(n+h+i_2)!(n+p+k+i_2)!(n+p+q+l+i_2)![(n+h)!]^{-1/2} \\ &\times \frac{1}{\sqrt{(n+p+k)!(n+p+q+l)!(n+h+i_2-i_1)(n+p+k+i_2-i_1)!(n+p+q+l+i_2-i_1)!}},\end{aligned} \tag{21}$$

where i_1 and i_2 are non-negative integers. In our numerical computation, factor $S_{Y;j}$ is always non-negative. For example, with fixed $p = q = 0$, $r = 4$, $h = k = l = 1$, we get $S_{Y;j} \approx 0.356$ (0.092, 0.011) as $j = 1$ (2, 3). Thus, the PATCS does not exist with the HOTMSS in $\hat{Y}$. Therefore, we expect that it will be revealed in $\hat{X}$. We use the analytic expression in Eq. (19) to clarify the property of the HOTMSS in the PATCS (Figure 3). It is shown that the PATCS exists with the HOTMSS in any orders. In addition, the negativity of $S_{X;j}$ becomes more obvious when order j decreases or/and parameter r increases. It is not difficult to see that the HOTMSS disappears in the small region of r. The numerical investigation indicates that the larger the photon-added number is, the higher the degree of HOTMSS will become. For example, when $p = q = 0$, $r = 8$ and $j = 2$, the degree of HOTMSS approaches 7, 11, 12, and 13% corresponding to $h = k = l = 1$, 2, 3, and 4, respectively.

Note that if $h + k + l$ is fixed, the degree of HOTMSS is the highest when $h = k = l$. For example, when $h + k + l = 6$, $p = q = 0$, $r = 8$ and $j = 2$, the degree of HOTMSS approaches 11, 10, and 9% corresponding to $(h, k, l) = (2, 2, 2)$, (4, 1, 1), and (6, 0, 0), respectively.

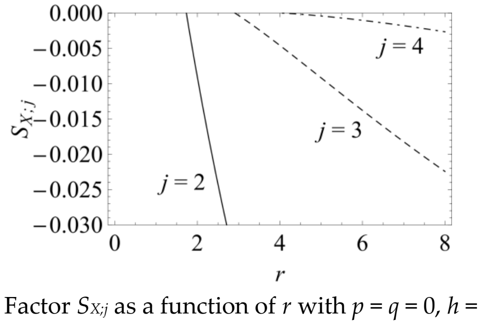

Fig. 3. Factor $S_{X;j}$ as a function of r with $p = q = 0$, $h = k = l = 2$ for $j = 2$ (the solid line), $j = 3$ (the dashed curve), and $j = 4$ (the dot-dashed curve)

4 Higher-order three-mode entanglement

Quantum entanglement plays a crucial role in the quantum information process. Recently, this property has been studied for quantum tasks such as quantum teleportation, quantum cryptography, quantum dense code, and quantum error correction [20]. Quantum entanglement only exists in multimode states and is detected by some criteria, for example, the Hillery–Zubairy criterion [21], the Shchukin–Vogel criterion [22]. In addition, there are several criteria for the detection of the entanglement degree, such as the von Neumann entropy [23], the linear entropy [24], and the concurrence [25]. In the three-mode case, the Duc et al. criterion [26] in the form of the inequality is given as

$$\begin{aligned}&\left|\langle\hat{a}^{m_a}\hat{b}^{m_b}\hat{c}^{m_c}\rangle\right| > \left(\langle\hat{a}^{+m_a}\hat{a}^{m_a}\rangle\langle\hat{b}^{+m_b}\hat{b}^{m_b}\rangle\langle\hat{c}^{+m_c}\hat{c}^{m_c}\rangle\right)^{1/2} \\ &= \left(\langle\hat{N}_a^{(m_a)}\rangle\langle\hat{N}_b^{(m_b)}\rangle\langle\hat{N}_c^{(m_c)}\rangle\right)^{1/2} \\ &\Leftrightarrow \left(\langle\hat{N}_a^{(m_a)}\rangle\langle\hat{N}_b^{(m_b)}\rangle\langle\hat{N}_c^{(m_c)}\rangle\right)^{1/2} - \left|\langle\hat{a}^{m_a}\hat{b}^{m_b}\hat{c}^{m_c}\rangle\right| < 0.\end{aligned} \tag{22}$$

We define the factor of higher-order three-mode entanglement (HOTME) as follows:

$$E = 1 - \frac{\left|\langle\hat{a}^{m_a}\hat{b}^{m_b}\hat{c}^{m_c}\rangle\right|}{\left(\langle\hat{N}_a^{(m_a)}\rangle\langle\hat{N}_b^{(m_b)}\rangle\langle\hat{N}_c^{(m_c)}\rangle\right)^{1/2}}. \tag{23}$$

A three-mode state is entangled in a higher-order if $E < 0$. Let us consider in the PATCS, if $m_a \neq m_b \neq m_c$, the quantum mean of operators $\hat{a}^{m_a}\hat{b}^{m_b}\hat{c}^{m_c}$ in this state is zero, thus the PATCS does not appear in the HOTME (obeying the above criterion). However, when $m_a = m_b = m_c = m$, from

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the analytical expression in Eq. (8), the factor of HOTME is given by

$$E_m = 1 - \frac{D_{m,0}}{(B_{m,0,0} B_{0,m,0} B_{0,0,m})^{1/2}}. \quad (24)$$

We use the analytical expression in Eq. (24) to evaluate the property of the HOTME in the PATCS. In Figure 4, we plot E_m as a function of r when $p = q = 0$, $h = k = l = 2$ for several values of m. The behaviour of the HOTME in the PATCS can be interpreted as follows: The higher value of r or/and m is/are, the more obvious the manifestation of the HOTME becomes. In addition, although the values of E_m decrease when increasing the number of photons added, the inequality in Eq. (22) is more violated. For example, when $m = 2$, $p = q = 0$, $r = 5$, the value of E_m achieves –0.79 (–0.46), corresponding to $h = k = l = 0$ ($h = k = l = 1$), but $(\langle \hat{N}_a^{(m)} \rangle \langle \hat{N}_b^{(m)} \rangle \langle \hat{N}_c^{(m)} \rangle)^{1/2} - |\langle \hat{a}^m \hat{b}^m \hat{c}^m \rangle|$ approaches –13.7 (–22.2). That means that the HOTME in the PATCS becomes more obvious when the number of photons added increases. On the other hand, if $h + k + l$ is fixed, E_m is minimal when $h = k = l$. For example, when $h + k + l = 6$, $p = q = 0$, $r = 5$, and $m = 2$, the values of E_m approach –0.43, –0.27, and –0.21 corresponding to $(h, k, l) = (6, 0, 0)$, $(4, 1, 1)$, and $(2, 2, 2)$, respectively.

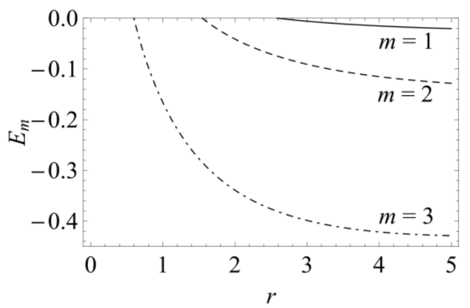

Fig. 4. Em as a fuction of r when $p = q = 0$ and $h = k = l = 2$ for $m = 1$ (the solid line), $m = 2$ (the dashed curve), and $m = 3$ (the dot-dashed curve)

5 Conclusions

In this paper, we investigated the higher-order nonclassical and entanglement properties in the PATCS, including the higher-order single-mode antibunching, the higher-order three-mode sum squeezing, and the higher-order three-mode entanglement. If the order is fixed, the role of photon addition operation in the PATCS is clearly exposed, in which the degree of the HOSMA increases, and the HOTMSS is improved by increasing the number of photons added to the TCS. Moreover, when the number of photons added to the TCS increases, the HOTME in the PATCS becomes more obvious. Therefore, the higher-order nonclassical and entanglement properties in the PATCS can be enhanced by local photon addition to the TCS. In addition, in the case of fixing the total of local photons added to the TCS, i.e., $h + k + l$ = constant, it is shown that when $h = k = l$, while HOTMSS is the most enhanced, and the HOSMA and the HOTME are least strengthened. If the order is changed, the degree of the HOSMA increases, and the HOTME is improved by increasing the values of the order. However, it is vice versa in the HOTMSS behaviour, i.e., the HOTMSS reduces when the values of the order increase.

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