Hue University Journal of Science: Natural Science
Vol. 130, No. 1D, 83–92, 2021

pISSN 1859-1388
eISSN 2615-9678

FINITE-REGION STABILITY OF 2-D SINGULAR ROESSER SYSTEMS WITH DIRECTIONAL DELAYS

Le Huy Vu[1], Le Anh Tuan[2]*

[1] Faculty of Natural Sciences, Hong Duc University, Thanh Hoa, Vietnam
[2] Faculty of Mathematics, University of Sciences, Hue University, 77 Nguyen Hue St., Hue, Vietnam

* Corresponding author: Le Anh Tuan <leanhtuan@hueuni.edu.vn>
(*Received: 9 April 2021; Accepted: 4 November 2021*)

Abstract. In this paper, the problem of finite-region stability is studied for a class of two-dimensional (2-D) singular systems described by Roesser model with directional delays. Based on the regularity, the underlying singular 2-D systems are first decomposed into fast- and slow-subsystems corresponding to dynamics and algebraic parts. Then, by virtue of Lyapunov-like 2-D functional method, we construct a weighted 2-D functional candidate and utilize zero-type free matrix equations to derive delay-dependent stability conditions in terms of linear matrix inequalities (LMIs). More specifically, the derived conditions ensure that all state trajectories of the system do not exceed a prescribed threshold over a pre-specified finite-region of times for any initial state sequences with energy-norms of dynamic parts do not exceed given bounds.

Keywords: Finite-region stability, 2-D singular systems, Roesser model, linear matrix inequalities

1 Introduction

Singular systems are widely used to describe dynamics of various practical phenomena such as electrical circuit networks, power systems, multibody mechanics, aerospace engineering and chemical and physical processes [1, 2, 3, 4]. In such a system, the state variables are subject to both dynamical equations and algebraic constraints which result a number of different features from classical systems such as impulsive behaviors in the state response, non-properness of transfer matrix or non-causality between input/output and states. These characteristic properties make the study of singular systems much more complicated and challenging than classical systems. On the other hand, as an inherent characteristic, time-delay is ubiquitously encountered in engineering systems which has various effects on the system performance [5, 6]. Thus, the study of qualitative behavior of time-delay systems plays an important role in applied models which has received significant research attention in the last two decades (see, e.g., [7, 8, 9] and the references therein). In particular, considerable effort from researchers has been devoted to the problems of stability analysis and control of singular systems with delays and many results have been reported in the literature. To mention a few, we refer the reader to [10, 11, 12] for the problem of stability analysis and [13, 14] for some other control issues related to singular delayed systems.

Two-dimensional (2-D) systems can be used to describe dynamics of many practical models where the information propagation occurs in each of the two independent directions [15, 16]. Recently, due to their widespread applications in circuit analysis, image processing, seismographic data transmission or multi-dimensional digital filtering, the theory of 2-D systems has attracted considerable research attention [17, 18, 19, 20]. There have been a few papers concerning the problems of stability and stabilization of 2-D descriptor systems. For example, in [21], the problems of stability and stabilization via state feedback controllers were investigated for a class of delay-free 2-D singular Roesser systems. By decomposing the system into slow- and fast-subsystems, and based on the Lyapunov function method, sufficient conditions in terms of linear matrix inequalities (LMIs) were derived to design a stabilizing state feedback controller. The problem of H_∞ control was also considered in [22, 23] for 2-D singular Roesser models with constant delays. By using the bounded real lemma approach, delay-independent LMI-based conditions were derived for the design of state feedback controllers that make the closed-loop system to be acceptable and stable with a prescribed H_∞ performance level. The problem of stability analysis was first extended for 2-D singular systems with generalized time-varying delays in [24]. Based on a 2-D Lyapunov–Krasovskii functional (LKF) scheme, and by employing a Jensen-type discrete inequality to manipulate the difference of a LKF candidate, delay-dependent stability conditions were derived subject to interval delays.

The concept of Lyapunov stability, recognized as infinite time behavior, has been well investigated and developed in the decades. However, in many practical applications, it is only required that the system states do not exceed a certain bound during a specified time interval for given bound on initial states. This gives rise to the use of finite-time stability (FTS) [25]. It is noted that, a system may be finite-time stable but not Lyapunov asymptotic stable and vice versa [25]. In 2-D systems, the information propagation occurs in each of the two independent directions. Thus, the concept of finite-region stability (FRS) can be regarded as a natural extension of FTS. The problem of stability analysis under FRS concept for 2-D systems has received considerably less attention and only a few results for 2-D systems have been reported [26, 27, 28]. However, the analysis and control involving FRS for 2-D singular systems has not been investigated in the literature. This motivates the present study.

In this paper, we consider the problem of finite-region stability of 2-D singular systems described by the Roesser model with directional state delays. First, based on the regularity, we decompose the underlying singular 2-D systems into fast- and slow-subsystems corresponding to dynamics and algebraic parts. Then, by utilizing zero-type free matrix equations techniques, delay-dependent conditions are derived in terms of linear matrix inequalities (LMIs) ensuring that state trajectories of the system do not exceed a prescribed threshold over a pre-specified finite-region of times.

Notation. $R^{n\times m}$ denotes the set of $n\times m$ matrices and $\mathrm{diag}(A,B)$ is the diagonal matrix formulated by stacking A and B. For a matrix $A\in\mathbb{R}^{n\times n}$, we let $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and minimal real part of eigenvalues of A. A matrix $M\in R^{n\times n}$ is semi-positive definite, $M\geq 0$, if $x^\top Mx\geq 0$ for all $x\in R^n$; M is positive definite, $M>0$, if $x^\top Mx>0$ for all $x\neq 0$. $S_n^+=\{M\in R^{n\times n}: M=M^\top>0\}$ and $S_m^+\oplus S_n^+=\left\{\mathrm{diag}(P^h,P^v): P^h\in S_m^+, P^v\in S_n^+\right\}$.

2 Preliminaries

Consider the following 2-D singular system described by the Roesser model with delays

$$E\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}=\mathcal{A}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}+\mathcal{A}_d\begin{bmatrix}x^h(i-d_h,j)\\ x^v(i,j-d_v)\end{bmatrix},\qquad (1)$$

where $x^h(i,j)\in R^{n_h}$ and $x^v(i,j)\in R^{n_v}$ are the horizontal and vertical state vectors, respectively. $\mathcal{A}=\begin{bmatrix}A^{hh} & A^{hv}\\ A^{vh} & A^{vv}\end{bmatrix}$ and $\mathcal{A}_d=\begin{bmatrix}A_d^{hh} & A_d^{hv}\\ A_d^{vh} & A_d^{vv}\end{bmatrix}$ are known real matrices. $E=\mathrm{diag}\{E_h,E_v\}$, where $E_h\in R^{n_h\times n_h}$

pISSN 1859-1388
eISSN 2615-9678

and $E_v \in R^{n_v \times n_v}$, is a singular matrix with $\mathrm{rank}(E) = r < n$, $\mathrm{rank}(E_h) = r_h \le n_h$ and $\mathrm{rank}(E_v) = r_v \le n_v$. d_h and d_v are positive scalars representing directional delays in the horizontal and vertical directions. Initial conditions of (1) are specified as

$$\begin{aligned} x^h(k,j) &= \phi(k,j),\ k \in Z[-d_h,0],\ j \ge 0, \\ x^v(i,l) &= \psi(i,l),\ l \in Z[-d_v,0],\ i \ge 0. \end{aligned} \tag{2}$$

For convenience, we denote

$$x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, x_d(i,j) = \begin{bmatrix} x^h(i-d_h,j) \\ x^v(i,j-d_v) \end{bmatrix}$$

as augmented state vectors of system (1). In addition, for given positive integers $D_1 \in N^+$ and $D_2 \in N^+$, we define the following rectangle finite-region

$$D_1 \times D_2 = \left\{(i,j) \in N_0^2 \mid 0 \le i \le D_1, 0 \le j \le D_2\right\}.$$

For a given symmetric semi-positive definite matrix W, we denote the weighted norms of ϕ and ψ as

$$\|\phi\|_{l_\infty}^W = \sup\left\{\phi^\top(k,j)W\phi(k,j) : -d_h \le k \le 0, j \ge 0\right\},$$

$$\|\psi\|_{l_\infty}^W = \sup\left\{\psi^\top(i,l)W\psi(i,l) : i \ge 0, -d_v \le l \le 0\right\}.$$

Let us first introduce the following definitions.

Definition 1 *System (1) is said to be (i) regular if the characteristic polynomial* $\det(EI(z,s) - A)$ *is not identically zero, where* $I(z,s) = \mathrm{diag}(zI_{n_h}, sI_{n_v})$*; and (ii) causal if* $\deg(\det(sE - A)) = \mathrm{rank}(E)$.

Definition 2 *Given positive scalars* c, c_h, c_v *and a matrix* $\Gamma \in S_{n_h}^+ \oplus S_{n_v}^+$. *System (1) is said to be singularly finite-region stable (SFRS) with respect to* $(c, c_h, c_v, D_1 \times D_2, \Gamma)$ *if for any initial sequences* ϕ, ψ *that satisfy* $\max\left\{\|\phi\|_{l_\infty}^{E_h^\top \Gamma_h E_h}, \|E_h\|^2 \|\phi\|_{l_\infty}^{\Gamma_h}\right\} < c_h$ *and* $\max\left\{\|\psi\|_{l_\infty}^{E_v^\top \Gamma_v E_v}, \|E_v\|^2 \|\psi\|_{l_\infty}^{\Gamma_v}\right\} < c_v$, *it holds that*

$$x^\top(i,j)E^\top \Gamma E x(i,j) < c \tag{3}$$

for all $(i,j) \in D_1 \times D_2$.

The main objective of this paper is to derive tractable LMI-based conditions by which 2-D singular systems in the form of (1) is regular, causal and SFRS.

3 Main results

In this section, we develop a 2-D Lyapunov-like functional method to derive SFRS for 2-D singular systems with delays in the form of (1). According to the singularity of system (1), we first decompose (1) into slow- and fast-subsystems. For this, let $\mathrm{rank}(E_h) = r_h \le n_h$ and $\mathrm{rank}(E_v) = r_v \le n_v$. Then, there always exist nonsingular matrices M^h, M^v, N^h and N^v such that

$$M^h E^h N^h = \mathrm{diag}\{I_{r_h}, 0\}, M^v E^v N^v = \mathrm{diag}\{I_{r_v}, 0\}. \tag{4}$$

By utilizing (4), we obtain the following decomposition

$$M^h A^{hh} N^h = \begin{bmatrix} A_{11}^{hh} & A_{12}^{hh} \\ A_{21}^{hh} & A_{22}^{hh} \end{bmatrix}, M^h A^{hv} N^v = \begin{bmatrix} A_{11}^{hv} & A_{12}^{hv} \\ A_{21}^{hv} & A_{22}^{hv} \end{bmatrix},$$

$$M^v A^{vv} N^v = \begin{bmatrix} A_{11}^{vv} & A_{12}^{vv} \\ A_{21}^{vv} & A_{22}^{vv} \end{bmatrix}, M^v A^{vh} N^h = \begin{bmatrix} A_{11}^{vh} & A_{12}^{vh} \\ A_{21}^{vh} & A_{22}^{vh} \end{bmatrix},$$

$$M^h A_d^{hh} N^h = \begin{bmatrix} A_{d11}^{hh} & A_{d12}^{hh} \\ A_{d21}^{hh} & A_{d22}^{hh} \end{bmatrix}, M^h A_d^{hv} N^v = \begin{bmatrix} A_{d11}^{hv} & A_{d12}^{hv} \\ A_{d21}^{hv} & A_{d22}^{hv} \end{bmatrix},$$

$$M^v A_d^{vv} N^v = \begin{bmatrix} A_{d11}^{vv} & A_{d12}^{vv} \\ A_{d21}^{vv} & A_{d22}^{vv} \end{bmatrix}, M^v A_d^{vh} N^h = \begin{bmatrix} A_{d11}^{vh} & A_{d12}^{vh} \\ A_{d21}^{vh} & A_{d22}^{vh} \end{bmatrix}.$$

The following lemma will be used to derive stability conditions in the section.

Lemma 1 *System (1) is regular and causal if the matrix* $A_{22} = \begin{bmatrix} A_{22}^{hh} & A_{22}^{hv} \\ A_{22}^{vh} & A_{22}^{vv} \end{bmatrix}$ *is nonsingular.*

Our main result is represented in the following theorem.

Theorem 1 *Assume that system (1) is regular and causal. Then, for given positive scalars c, c_h, c_v and a matrix $\Gamma \in S^+_{n_h} \oplus S^+_{n_v}$, system (1) is SFRS with respect to $(c, c_h, c_v, D_1 \times D_2, \Gamma)$ if there exist positive scalars α_h, α_v, β_h, β_v, μ_h, μ_v, $\gamma \in (0,1)$, symmetric positive-definite matrices P, Q, R, S in $S^+_{n_h} \oplus S^+_{n_v}$ and matrices Y^{hh}, Y^{vv}, Z^{vh}, Z^{hv}, H^{vh}, H^{hv} of appropriate dimensions that satisfy the following conditions*

$$\Phi = \left(\Phi_{kl}\right)_{k,l=1}^{4} < 0, \quad \Psi = \left(\Psi_{kl}\right)_{k,l=1}^{4} < 0 \tag{5}$$

$$c_h < \gamma c, \quad c_v < (1-\gamma)c \tag{6}$$

$$\lambda^+(\overline{P}^h) + \frac{\lambda^+(\overline{Q}^h) s_{\alpha_h}}{\|E_h\|^2} + \beta_h (1-\gamma) D_1 \lambda^+(\overline{R}^v) + \mu_h (1-\gamma) D_1 \lambda^+(\overline{S}^v) < \frac{\gamma c \lambda_+(\overline{P}^h)}{c_h \overline{\alpha}} \tag{7}$$

$$\lambda^+(\overline{P}^v) + \frac{\lambda^+(\overline{Q}^v) s_{\alpha_v}}{\|E_v\|^2} + \beta_v \gamma D_2 \lambda^+(\overline{R}^h) + \mu_v \gamma D_2 \lambda^+(\overline{S}^h) < \frac{(1-\gamma) c \lambda_+(\overline{P}^v)}{c_v \overline{\alpha}} \tag{8}$$

where, for $M \in S^+_p$, we denote $\lambda^+(M) = \lambda_{\max}(M)$, $\lambda_+(M) = \lambda_{\min}(M)$, $\overline{\alpha} = \max\left\{1, \alpha_h^{D_1}, \alpha_v^{D_2}\right\}$ and other notations are defined as follows

$$s_{\alpha_h} = \sum_{k=-d_h}^{-1} \alpha_h^{-(k+1)}, \quad s_{\alpha_v} = \sum_{l=-d_v}^{-1} \alpha_v^{-(l+1)},$$

$$\overline{P} = \Gamma^{-\frac{1}{2}} P \Gamma^{-\frac{1}{2}}, \overline{Q} = \Gamma^{-\frac{1}{2}} Q \Gamma^{-\frac{1}{2}}, \overline{R} = \Gamma^{-\frac{1}{2}} R \Gamma^{-\frac{1}{2}},$$

$$\overline{S} = \Gamma^{-\frac{1}{2}} S \Gamma^{-\frac{1}{2}}, X^h = (E_h^\top)^\perp, X^v = (E_v^\top)^\perp,$$

$$\Psi_{11} = (A^{hh})^\top P^h A^{hh} + Q^h - Y^{hh} (X^h)^\top A^{hh} - (A^{hh})^\top X^h (Y^{hh})^\top - \alpha_h (E_h)^\top P^h E_h,$$

$$\Psi_{12} = (A^{hh})^\top P^h A^{hv} - Y^{hh} (X^h)^\top A^{hv} - (A^{hh})^\top X^h (H^{vh})^\top,$$

$$\Psi_{13} = (A^{hh})^\top P^h A_d^{hh} - Y^{hh} (X^h)^\top A_d^{hh},$$

$$\Psi_{14} = (A^{hh})^\top P^h A_d^{hv} - Y^{hh} (X^h)^\top A_d^{hv} - (A^{hh})^\top X^h (Z^{vh})^\top,$$

$$\Psi_{22} = (A^{hv})^\top P^h A^{hv} - \frac{\beta_h c_h}{c} (E_v)^\top R^v E_v - H^{vh} (X^h)^\top A^{hv} - (A^{hv})^\top X^h (H^{vh})^\top,$$

$$\Psi_{23} = (A^{hv})^\top P^h A_d^{hh} - H^{vh} (X^h)^\top A_d^{hh},$$

$$\Psi_{24} = (A^{hv})^\top P^h A_d^{hv} - H^{vh} (X^h)^\top A_d^{hv} - (A^{hv})^\top X^h (Z^{vh})^\top,$$

$$\Psi_{33} = (A_d^{hh})^\top P^h A_d^{hh} - \alpha_h^{d_h} Q^h,$$

$$\Psi_{34} = (A_d^{hh})^\top P^h A_d^{hv} - (A_d^{hh})^\top X^h (Z^{vh})^\top,$$

$$\Psi_{44} = (A_d^{hv})^\top P^h A_d^{hv} - \frac{\mu_h c_h}{c} (E^v)^\top S^v E^v - Z^{vh} (X^h)^\top A_d^{hv} - (A_d^{hv})^\top X^h (Z^{vh})^\top,$$

$$\Phi_{11} = (A^{vh})^\top P^v A^{vh} - \frac{\beta_v c_v}{c} (E_h)^\top R^h E_h - H^{hv} (X^v)^\top A^{vh} - (A^{vh})^\top X^v (H^{hv})^\top,$$

$$\Phi_{12} = (A^{vh})^\top P^v A^{vv} - (A^{vh})^\top X^v (Y^{vv})^\top - H^{hv} (X^v)^\top A^{vv},$$

$$\Phi_{13} = (A^{vh})^\top P^v A_d^{vh} - H^{hv} (X^v)^\top A_d^{vh} - (A^{vh})^\top X^v (Z^{hv})^\top,$$

$$\Phi_{14} = (A^{vh})^\top P^v A_d^{vv} - H^{hv} (X^v)^\top A_d^{vv},$$

$$\Phi_{22} = (A^{vv})^\top P^v A^{vv} + Q^v - Y^{vv} (X^v)^\top A^{vv} - (A^{vv})^\top X^v (Y^{vv})^\top - \alpha_v (E^v)^\top P^v E^v,$$

$$\Phi_{23} = (A^{vv})^\top P^v A_d^{vh} - Y^{vv} (X^v)^\top A_d^{vh} - (A^{vv})^\top X^v (Z^{hv})^\top,$$

$$\Phi_{24} = (A^{vv})^\top P^v A_d^{vv} - Y^{vv} (X^v)^\top A_d^{vv},$$

$$\Phi_{33} = (A_d^{vh})^\top P^v A_d^{vh} - \frac{\mu_v c_v}{c} (E_h)^\top S^h E_h - Z^{hv} (X^v)^\top A_d^{vh} - (A_d^{vh})^\top X^v (Z^{hv})^\top,$$

$$\Phi_{34} = (A_d^{vh})^\top P^v A_d^{vv} - Z^{hv} (X^v)^\top A_d^{vv},$$

$$\Phi_{44} = (A_d^{vv})^\top P^v A_d^{vv} - \alpha_v^{d_v} Q^v.$$

Proof. By virtue of the 2-D Lyapunov-Krasovskii functional method, we construct the following functional candidate

$$V^h(x^h(i,j)) = x^{h\top}(i,j) E_h^\top P^h E_h x^h(i,j) + \sum_{k=i-d_h}^{i-1} \alpha_h^{i-1-k} x^{h\top}(k,j) Q^h x^h(k,j), \tag{9}$$

$$V^v(x^v(i,j)) = x^{v\top}(i,j) E_v^\top P^v E_v x^v(i,j) + \sum_{l=j-d_v}^{j-1} \alpha_v^{j-1-l} x^{v\top}(i,l) Q^v x^v(i,l). \tag{10}$$

pISSN 1859-1388
eISSN 2615-9678

We first compute the difference of $V^h(x^h(i,j))$ along state trajectories of system (1). According to (9) and (1), we have

$$\Delta_h(i,j) \triangleq V^h(x^h(i+1,j)) - \alpha_h V^h(x^h(i,j))$$

$$-\frac{\beta_h c_h}{c} x^{v\top}(i,j)E_v^\top R^v E_v x^v(i,j)$$

$$-\frac{\mu_h c_h}{c} x^{v\top}(i,j-d_v)E_v^\top S^v E_v x^v(i,j-d_v)$$

$$= x^{h\top}(i+1,j)E_h^\top P^h E_h x^h(i+1,j)$$

$$+\sum_{k=i+1-d_h}^{i} \alpha_h^{i-k} x^{h\top}(k,j)Q^h x^h(k,j)$$

$$-\alpha_h x^{h\top}(i,j)E_h^\top P^h E_h x^h(i,j)$$

$$-\sum_{k=i-d_h}^{i-1} \alpha_h^{i-k} x^{h\top}(k,j)Q^h x^h(k,j)$$

$$-\frac{\beta_h c_h}{c} x^{v\top}(i,j)E_v^\top R^v E_v x^v(i,j)$$

$$-\frac{\mu_h c_h}{c} x^{v\top}(i,j-d_v)E_v^\top S^v E_v x^v(i,j-d_v).$$

Therefore,

$$\begin{aligned}\Delta_h(i,j) = &\left(\mathcal{A}_h x(i,j) + \mathcal{A}_{dh} x_d(i,j)\right)^\top P^h \\ &\times\left(\mathcal{A}_h x(i,j) + \mathcal{A}_{dh} x_d(i,j)\right) + x^{h\top}(i,j)Q^h x^h(i,j) \\ &-\alpha_h^{d_h} x^{h\top}(i-d_h,j)Q^h x^h(i-d_h,j) \\ &-\alpha_h x^{h\top}(i,j)E_h^\top P^h E_h x^h(i,j) \\ &-\frac{\beta_h c_h}{c} x^{v\top}(i,j)E_v^\top R^v E_v x^v(i,j) \\ &-\frac{\mu_h c_h}{c} x^{v\top}(i,j-d_v)E_v^\top S^v E_v x^v(i,j-d_v),\end{aligned} \tag{11}$$

where $\mathcal{A}_h = [A^{hh} \quad A^{hv}]$ and $\mathcal{A}_{dh} = [A_d^{hh} \quad A_d^{hv}]$.

On the other hand, since X^h is the null space of the matrix $E_h^\top$, we have $E_h^\top X^h = 0$. Thus, for any matrices Y^{hh}, H^{vh} and Z^{vh} of appropriate dimensions, the following zero-type equation holds

$$\begin{aligned}-2[x^{h\top}(i,j)Y^{hh} &+ x^{v\top}(i,j)H^{vh} + x^{v\top}(i,j-d_v)Z^{vh}] \\ &\times X^{h\top}E_h x^h(i+1,j) = 0.\end{aligned} \tag{12}$$

It follows from (1) and (12) that

$$\begin{aligned}-2[x^{h\top}(i,j)Y^{hh} &+ x^{v\top}(i,j)H^{vh} + x^{v\top}(i,j-d_v)Z^{vh}] \\ &\times X^{h\top}\left(\mathcal{A}_h x(i,j) + \mathcal{A}_{dh} x_d(i,j)\right) = 0.\end{aligned} \tag{13}$$

Combining (11) and (13), we obtain

$$\Delta_h(i,j) = \xi^\top(i,j)\Psi\xi(i,j) \tag{14}$$

where $\xi(i,j) = [x^\top(i,j) \quad x_d^\top(i,j)]^\top$. By (5), $\Psi < 0$, and hence $\Delta_h(i,j) \le 0$, which, together with (14), yields

$$\begin{aligned}V^h(x^h(i+1,j)) &\le \alpha_h V^h(x^h(i,j)) \\ &+\frac{\beta_h c_h}{c} x^{v\top}(i,j)E_v^\top R^v E_v x^v(i,j) \\ &+\frac{\mu_h c_h}{c} x^{v\top}(i,j-d_v)E_v^\top S^v E_v x^v(i,j-d_v).\end{aligned} \tag{15}$$

By induction, from (15) we readily obtain

$$\begin{aligned}V^h(x^h(i,j)) &< \alpha_h^i V^h(x^h(0,j)) \\ &+\sum_{k=0}^{i-1} \alpha_h^{i-1-k}\Big[\frac{\beta_h c_h}{c} x^{v\top}(k,j)E_v^\top R^v E_v x^v(k,j) \\ &+\frac{\mu_h c_h}{c} x^{v\top}(k,j-d_v)E_v^\top S^v E_v x^v(k,j-d_v)\Big].\end{aligned} \tag{16}$$

In addition to this, from (9), we have

$$\begin{aligned}V^h(x^h(i,j)) &> x^{h\top}(i,j)E_h^\top P^h E_h x^h(i,j) \\ &= x^{h\top}(i,j)E_h^\top (\Gamma^h)^{\frac{1}{2}} \overline{P}^h (\Gamma^h)^{\frac{1}{2}} E_h x^h(i,j) \\ &\ge \lambda_+(\overline{P}^h) x^{h\top}(i,j)E_h^\top \Gamma^h E_h x^h(i,j).\end{aligned} \tag{17}$$

We now estimate the value of $V^h(x^h(0,j))$. For this, we notice that

$$V^h(x^h(0,j)) = x^{h\top}(0,j)E_h^\top P^h E_h x^h(0,j)$$

$$+\sum_{k=-d_h}^{-1} \alpha_h^{-1-k} x^{h\top}(k,j)Q^h x^h(k,j)$$

$$= x^{h\top}(0,j)E_h^\top (\Gamma^h)^{\frac{1}{2}} \overline{P}^h (\Gamma^h)^{\frac{1}{2}} E_h x^h(0,j)$$

$$+\sum_{k=-d_h}^{-1} \alpha_h^{-1-k} x^{h\top}(k,j)(\Gamma^h)^{\frac{1}{2}} \overline{Q}^h (\Gamma^h)^{\frac{1}{2}} x^h(k,j).$$

Therefore,

$$\begin{aligned}V^h(x^h(0,j)) \le\ &\lambda^+(\overline{P}^h) x^{h\top}(0,j)E_h^\top \Gamma^h E_h x^h(0,j) \\ &+\lambda^+(\overline{Q}^h) s_{\alpha_h} \|\phi\|_{\infty}^{\Gamma^h}.\end{aligned} \tag{18}$$

For initial sequences (ϕ,ψ), assume that

$$\max\left\{\|\phi\|_{l_\infty}^{E_h^\top\Gamma_h E_h}, \|E_h\|^2\|\phi\|_{l_\infty}^{\Gamma_h}\right\} < c_h.$$

Then, we have

$$V^h(x^h(0,j)) \le \left(\lambda^+(\overline{P}^h) + \frac{\lambda^+(\overline{Q}^h)s_{\alpha_h}}{\|E_h\|^2}\right)c_h. \tag{19}$$

The next two terms in (16) can be manipulated as follows

$$\begin{aligned}&\sum_{k=0}^{i-1}\frac{\beta_h c_h}{c}\alpha_h^{i-1-k}x^{v\top}(k,j)E_v^\top R^v E_v x^v(k,j)\\ &= \frac{\beta_h c_h}{c}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j)E_v^\top(\Gamma^v)^{\frac{1}{2}}\overline{R}^v(\Gamma^v)^{\frac{1}{2}}E_v x^v(k,j)\\ &\le \frac{\beta_h c_h\lambda^+(\overline{R}^v)}{c}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j)E_v^\top\Gamma^v E_v x^v(k,j)\end{aligned} \tag{20}$$

and

$$\begin{aligned}&\sum_{k=0}^{i-1}\frac{\mu_h c_h}{c}\quad\alpha_h^{i-1-k}x^{v\top}(k,j-d_v)E_v^\top S_v E_v x^v(k,j-d_v)\\ &\quad= \frac{\mu_h c_h}{c}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j-d_v)E_v^\top(\Gamma^v)^{\frac{1}{2}}\\ &\quad\quad\times\overline{S}^v(\Gamma^v)^{\frac{1}{2}}E_v x^v(k,j-d_v)\\ &\quad\le \frac{\mu_h c_h\lambda^+(\overline{S}^v)}{c}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j-d_v)\\ &\quad\quad\times E_v^\top\Gamma^v E_v x^v(k,j-d_v).\end{aligned} \tag{21}$$

From (16) to (21), we finally obtain

$$\begin{aligned}&x^{h\top}(i,j)E_h^\top\Gamma^h E_h x^h(i,j)\\ &< \frac{c_h\overline{\alpha}}{\lambda_+(\overline{P}^h)}\left(\lambda^+(\overline{P}^h) + \frac{\lambda^+(\overline{Q}^h)s_{\alpha_h}}{\|E_h\|^2}\right)\\ &+\frac{\beta_h c_h\lambda^+(\overline{R}^v)}{c\lambda_+(\overline{P}^h)}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j)E_v^\top\Gamma^v E_v x^v(k,j)\\ &+\frac{\mu_h c_h\lambda^+(\overline{S}^v)}{c\lambda_+(\overline{P}^h)}\sum_{k=0}^{i-1}\alpha_h^{i-1-k}x^{v\top}(k,j-d_v)\\ &\quad\times E_v^\top\Gamma^v E_v x^v(k,j-d_v).\end{aligned} \tag{22}$$

By similar lines used in formulating (22), we then obtain

$$\begin{aligned}&x^{v\top}(i,j)E_v^\top\Gamma^v E_v x^v(i,j)\\ &< \frac{c_v\overline{\alpha}}{\lambda_+(\overline{P}^v)}\left(\lambda^+(\overline{P}^v) + \frac{\lambda^+(\overline{Q}^v)s_{\alpha_v}}{\|E_v\|^2}\right)\\ &+\frac{\beta_v c_v\lambda^+(\overline{R}^h)}{c\lambda_+(\overline{P}^v)}\sum_{l=0}^{j-1}\alpha_v^{j-1-l}x^{h\top}(i,l)E_h^\top\Gamma^h E_h x^h(i,l)\\ &+\frac{\mu_v c_v\lambda^+(\overline{S}^h)}{c\lambda_+(\overline{P}^v)}\sum_{l=0}^{j-1}\alpha_v^{j-1-l}x^{h\top}(i-d_h,l)\\ &\quad\times E_h^\top\Gamma^h E_h x^h(i-d_h,l).\end{aligned} \tag{23}$$

We now prove by induction in i that, for any j satisfying $0 \le j \le D_2$, by condition (6), the following inequalities hold

$$x^{h\top}(i,j)E_h^\top\Gamma^h E_h x^h(i,j) < \gamma c \tag{24}$$

and

$$x^{v\top}(i,j)E_v^\top\Gamma^v E_v x^v(i,j) < (1-\gamma)c. \tag{25}$$

Indeed, let $i = 0$ in (23). For $j = 0,1,\ldots,D_2-1$, we have

$$\begin{aligned}&x^{v\top}(0,j)E_v^\top\Gamma^v E_v x^v(0,j)\\ &< \frac{c_v\overline{\alpha}}{\lambda_+(\overline{P}^v)}\left(\lambda^+(\overline{P}^v) + \frac{\lambda^+(\overline{Q}^v)s_{\alpha_v}}{\|E_v\|^2}\right)\\ &+\frac{c_v\overline{\alpha}D_2}{c\lambda_+(\overline{P}^v)}\left(\beta_v\lambda^+(\overline{R}^h) + \mu_v\lambda^+(\overline{S}^h)\right)\gamma c.\end{aligned} \tag{26}$$

In view of inequality (26) and condition (8), we arrive at

$$x^{v\top}(0,j)E_v^\top\Gamma^v E_v x^v(0,j) < (1-\gamma)c. \tag{27}$$

Let $i = 1$ in (22), for $0 \le j \le D_2$, we have

$$\begin{aligned}&x^{h\top}(1,j)E_h^\top\Gamma^h E_h x^h(1,j)\\ &< \frac{c_h\overline{\alpha}}{\lambda_+(\overline{P}^h)}\left(\lambda^+(\overline{P}^h) + \frac{\lambda^+(\overline{Q}^h)s_{\alpha_h}}{\|E_h\|^2}\right)\\ &+\frac{c_h}{c\lambda_+(\overline{P}^h)}[\beta_h\lambda^+(\overline{R}^v)x^{v\top}(0,j)E_v^\top\Gamma^v E_v x^v(0,j)\\ &+\mu_h\lambda^+(\overline{S}^v)x^{v\top}(0,j-d_v)E_v^\top\Gamma^v E_v x^v(0,j-d_v)].\end{aligned} \tag{28}$$

If $0 \le j \le d_v$, then $l = j - d_v \in [-d_v, 0]$ and we have

$$x^{v\top}(0,j-d_v)E_v^\top\Gamma^v E_v x^v(0,j-d_v)$$

pISSN 1859-1388
eISSN 2615-9678

$$= \psi^\top(0,l)E_v^\top \Gamma^v E_v \psi(0,l)$$

$$\leq \|\psi\|_{l_\infty}^{E_v^\top \Gamma^v E_v} < c_v < (1-\gamma)c.$$

If $j > d_v$ then $j - d_v > 0$. By (27), we have

$$x^{v\top}(0, j-d_v)E_v^\top \Gamma^v E_v x^v(0, j-d_v) < (1-\gamma)c.$$

Therefore,

$$x^{v\top}(0, j-d_v)E_v^\top \Gamma^v E_v x^v(0, j-d_v) < (1-\gamma)c \qquad (29)$$

holds for all $j \geq 0$. From (27)-(29), we obtain

$$\begin{aligned} & x^{h\top}(1,j)E_h^\top \Gamma^h E_h x^h(1,j) \\ & < \frac{c_h \bar{\alpha}}{\lambda_+(\bar{P}^h)} \left(\lambda^+(\bar{P}^h) + \frac{\lambda^+(\bar{Q}^h) s_{\alpha_h}}{\|E_h\|^2} \right) \\ & + \frac{D_1 c_h \bar{\alpha}}{\lambda_+(\bar{P}^h)} \left(\beta_h \lambda^+(\bar{R}^v) + \mu_h \lambda^+(\bar{S}^v) \right)(1-\gamma). \end{aligned} \qquad (30)$$

By condition (7), the estimation (30) gives

$$x^{h\top}(1,j)E_h^\top \Gamma^h E_h x^h(1,j) < \gamma c. \qquad (31)$$

Let $i = 1$ in (23), we have

$$\begin{aligned} & x^{v\top}(1,j)E_v^\top \Gamma^v E_v x^v(1,j) \\ & < \frac{c_v \bar{\alpha}}{\lambda_+(\bar{P}^v)} \left(\lambda^+(\bar{P}^v) + \frac{\lambda^+(\bar{Q}^v) s_{\alpha_v}}{\|E_v\|^2} \right) \\ & + \frac{\beta_v c_v \lambda^+(\bar{R}^h)}{c\lambda_+(\bar{P}^v)} \sum_{l=0}^{j-1} \alpha_v^{j-1-l} x^{h\top}(1,l)E_h^\top \Gamma^h E_h x^h(1,l) \\ & + \frac{\mu_v c_v \lambda^+(\bar{S}^h)}{c\lambda_+(\bar{P}^v)} \sum_{l=0}^{j-1} \alpha_v^{j-1-l} x^{h\top}(1-d_h,l) \\ & \times E_h^\top \Gamma^h E_h x^h(1-d_h,l). \end{aligned} \qquad (32)$$

If $d_h = 0$ then by utilizing (31), we have

$$x^{h\top}(1-d_h,l)E_h^\top \Gamma^h E_h x^h(1-d_h,l)$$

$$= x^{h\top}(1,l)E_h^\top \Gamma^h E_h x^h(1,l)$$

$$< \gamma c.$$

If $d_h > 0$, then $k = 1 - d_h \in [-d_h, 0]$. Therefore,

$$x^{h\top}(1-d_h,l)E_h^\top \Gamma^h E_h x^h(1-d_h,l)$$

$$= \phi^\top(k,l)E_h^\top \Gamma^h E_h \phi(k,l)$$

$$\leq \|\phi\|_{l_\infty}^{E_h^\top \Gamma^h E_h}$$

$$< c_h < \gamma c.$$

As a consequence,

$$x^{h\top}(1-d_h,l)E_h^\top \Gamma^h E_h x^h(1-d_h,l) < \gamma c. \qquad (33)$$

It follows from (31)-(33) and condition (8) that

$$\begin{aligned} & x^{v\top}(1,j)E_v^\top \Gamma^v E_v x^v(1,j) \\ & < \frac{c_v \bar{\alpha}}{\lambda_+(\bar{P}^v)} \left(\lambda^+(\bar{P}^v) + \frac{\lambda^+(\bar{Q}^v) s_{\alpha_v}}{\|E_v\|^2} \right) \\ & + \frac{c_v \bar{\alpha} D_2}{\lambda_+(\bar{P}^v)} \left(\beta_v \lambda^+(\bar{R}^h) + \mu_v \lambda^+(\bar{S}^h) \right)\gamma \\ & < (1-\gamma)c. \end{aligned} \qquad (34)$$

As a final step, assume that estimate (24) holds for $0 \leq i \leq D_1 - 1$, that is,

$$x^{h\top}(i,j)E_h^\top \Gamma^h E_h x^h(i,j) < \gamma c,\ i = 0,1,\ldots,D_1 - 1.$$

then, it is clear from (23) that

$$x^{v\top}(i,j)E_v^\top \Gamma^v E_v x^v(i,j) < (1-\gamma)c \qquad (35)$$

holds for $i = 0,1,\ldots,D_1 - 1$. Now, for $i = D_1$, $0 \leq j \leq D_2$, from (22), we have

$$\begin{aligned} & x^{h\top}(D_1,j)E_h^\top \Gamma^h E_h x^h(D_1,j) \\ & < \frac{c_h \bar{\alpha}}{\lambda_+(\bar{P}^h)} \left(\lambda^+(\bar{P}^h) + \frac{\lambda^+(\bar{Q}^h) s_{\alpha_h}}{\|E_h\|^2} \right) \\ & + \frac{\beta_h c_h \lambda^+(\bar{R}^v)}{c\lambda_+(\bar{P}^h)} \sum_{k=0}^{D_1-1} \alpha_h^{D_1-1-k} x^{v\top}(k,j)E_v^\top \Gamma^v E_v x^v(k,j) \\ & + \frac{\mu_h c_h \lambda^+(\bar{S}^v)}{c\lambda_+(\bar{P}^h)} \sum_{k=0}^{D_1-1} \alpha_h^{D_1-1-k} x^{v\top}(k, j-d_v) \\ & \times E_v^\top \Gamma^v E_v x^v(k, j-d_v). \end{aligned} \qquad (36)$$

Note that, if $0 \leq j \leq d_v$, then $j - d_v \in [-d_v, 0]$ and we have

$$x^{v\top}(i, j-d_v)E_v^\top \Gamma^v E_v x^v(i, j-d_v) < c_v < (1-\gamma)c$$

for all $i \leq D_1 - 1$. If $j > d_v$, then $j - d_v > 0$. From (35),

$$x^{v\top}(i, j-d_v)E_v^\top \Gamma^v E_v x^v(i, j-d_v) < (1-\gamma)c$$

for $0 \leq i \leq D_1 - 1$. Therefore,

$$x^{v\top}(i, j-d_v)E_v^\top \Gamma^v E_v x^v(i, j-d_v) < (1-\gamma)c, \qquad (37)$$

for all $0 \le i \le D_1 - 1, j \ge 0$. This, by condition (7), shows that

$$\begin{aligned} & x^h(D_1,j)E_h^\top L^h E_h x^h(D_1,j) \\ & < \frac{c_h\bar{\alpha}}{\lambda_+(\bar{P}^h)}\left(\lambda^+(\bar{P}^h) + \frac{\lambda^+(\bar{Q}^h)s_{\alpha_h}}{\|E_h\|^2}\right) \\ & + \frac{D_1 c_h \bar{\alpha}}{\lambda_+(\bar{P}^h)}\left(\beta_h\lambda^+(\bar{R}^v) + \mu_h\lambda^+(\bar{S}^v)\right)(1-\gamma) \\ & < \gamma c. \end{aligned} \tag{38}$$

By the same arguments used in deriving (38), for $i = D_1$, $0 \le j \le D_2$, we also have

$$x^{v\top}(D_1,j)E_v^\top \Gamma^v E_v x^v(D_1,j) < (1-\gamma)c. \tag{39}$$

Thus, for any $(i,j) \in D_1 \times D_2$, it follows from (38) and (39) that

$$\begin{aligned} x^\top(i,j)E^\top\Gamma E x(i,j) = \; & x^{h\top}(i,j)E_h^\top \Gamma^h E_h x^h(i,j) \\ & + x^{v\top}(i,j)E_v^\top\Gamma^v E_v x^v(i,j) \\ & < \gamma c + (1-\gamma)c = c. \end{aligned} \tag{40}$$

The last inequality in (40) shows that the 2-D singular systems in the form of (1) is SFRS with respect to $(c, c_h, c_v, D_1 \times D_2, \Gamma)$. The proof is completed.

Remark 1 *For given positive scalars c, c_h, c_v and positive integers D_1, D_2, the derived stability conditions in Theorem 1 are still involved non-convex scalars α_h, α_v and therefore $\bar{\alpha}$. However, by fixing the scalars α_h, α_v, the derived conditions in (7)-(8) can be reduced to LMIs, which, together with (5)-(6), can be effectively solved by various convex algorithms, for instance, the interior-point algorithm implemented in Matlab LMI Control Toolbox. In addition, by iteratively solving the LMIs given in Theorem 1 with respect to turning parameter c for fixed parameters c_h, c_v and D_1, D_2, we can find a possible minimum bound of the threshold c.*

An illustrative example

Consider a thermal process which is described by the following delayed partial differential equation

$$\frac{\partial T(x,t)}{\partial x} + \frac{\partial T(x,t)}{\partial t} = a_0 T(x,t) + a_1 T(x-\tau_x,t) + a_2 T(x,t-\tau_t), \tag{41}$$

where $T(x,t)$ is an unknown function like temperature, for example, at space $x \in [0,L]$ and time $t \in [0,\infty)$, a_0, a_1, a_2 are real constant coefficients and τ_x, τ_t are time-varying delays.

For given increments Δx and Δt, denote $T(i,j) = T(i\Delta x, j\Delta t)$ and the derivatives $\frac{\partial T(x,t)}{\partial x}$, $\frac{\partial T(x,t)}{\partial t}$ will be computed by using the forward Euler scheme as

$$\frac{\partial T(x,t)}{\partial x} \simeq \frac{T(i,j) - T(i-1,j)}{\Delta x},$$

$$\frac{\partial T(x,t)}{\partial t} \simeq \frac{T(i,j) - T(i,j-1)}{\Delta t}.$$

Define $x^h(i,j) = T(i-1,j)$ and $x^v(i,j) = T(i,j)$ then the corresponding discretization process transforms (41) into a 2-D discrete-time system of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \mathcal{A}\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \mathcal{A}_d\begin{bmatrix} x^h(i-\tau_h(i),j) \\ x^v(i,j-\tau_v(j)) \end{bmatrix}, \tag{42}$$

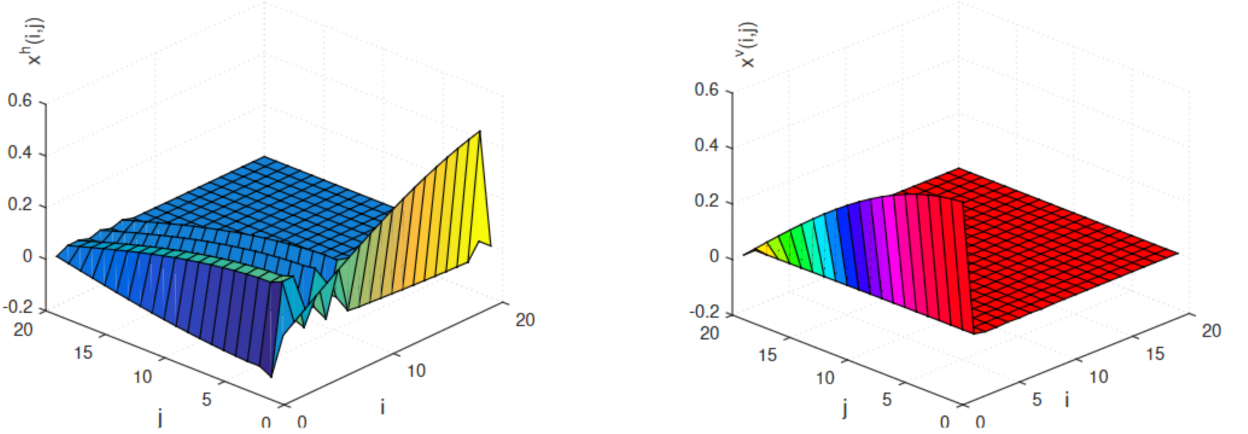
where $\tau_h(i) = \lfloor \tau_x / \Delta x \rfloor$ and $\tau_v(j) = \lfloor \tau_t / \Delta t \rfloor$.

For illustrative purpose, the system matrices of (42) are given as

$$\mathcal{A} = \begin{bmatrix} -0.084 & -0.7186 \\ 0.084 & 1.7186 \end{bmatrix}, \quad \mathcal{A}_d = \begin{bmatrix} 0 & 0.4286 \\ 0 & -0.4286 \end{bmatrix}.$$

Hue University Journal of Science: Natural Science
Vol. 130, No. 1D, 83–92, 2021

pISSN 1859-1388
eISSN 2615-9678



(a) $x^h(i,j)$ (b) $x^v(i,j)$

Fig. 1. A state trajectory of system (38) with $\tau_h(i) = 1 + 4|\sin(\pi i/2)|$, $\tau_v(j) = 1 + 5|\sin(\pi j/2)|$, $d_h = 5, d_v = 6$

With delays $1 \le \tau_h(i) \le 5$, $1 \le \tau_v(j) \le 6$, $\Gamma = I_2$, and for given scalars $c_h = c_v = 0.5$, $D_1 = D_2 = 20$, by using Matlab LMI toolbox, it is found that the derived conditions in Theorem 1 are satisfied with $c = 5$ (other parameters are omitted here). By Theorem 1, system (42) is singularly finite-region stable with respect to $(c, c_h, c_v, D_1 \times D_2, \Gamma)$. Simulation results presented in Fig. 1(a)-(b) are taken with $\tau_h(i) = 1 + 4|\sin(\pi i/2)|$, $\tau_v(j) = 1 + 5|\sin(\pi j/2)|$ and finite support initial sequences $x^h(k,j) = 0.5$ for $k \in Z[-5,0]$, $j \le 30$, and $x^v(i,l) = 0.5$ for $i \le 30, l \in Z[-6,0]$. It can be seen that the corresponding state trajectory of (42) is confined within the threshold as revealed by the result of Theorem 1.

4 Conclusion

The problem of finite-region stability has been studied for a class of 2-D singular systems in the Roesser model with directional delays. Based on a 2-D Lyapunov-like functional method and zero-type free matrix equations techniques, delay-dependent finite-region stability conditions have been derived in terms of tractable LMIs. More specifically, it has been shown that, by the derived LMI-based conditions, all state trajectories of the system do not exceed a prescribed threshold over a pre-specified finite-region of times for any initial state sequences with energy-norms of dynamic parts do not exceed given bounds.

Funding statement

This work is supported by Hue University under Grant No DHH2021-01-188.

References

1. Duan GR. Analysis and Design of Descriptor Linear Systems. New York: Springer; 2010.
2. Xu S, Lam J. Robust Control and Filtering of Singular Systems. New York: Springer; 2006.
3. Aplevich JD. Implicit Linear Systems. Berlin: Springer-Varlag; 1991.
4. Emmrich E, Mehrmann V. Operator differential-algebraic equations arising in fluid dynamics. Comput Method Appl Math 2013;13:443-470.
5. Insperger T, Ersal T, Orosz G. Time Delay Systems: Theory, Numerics, Applications, and Experiments. Switzerland: Springer; 2017.
6. Witrant E, Fridman E, Sename O, Dugard L. Recent Results on Time-Delay Systems: Analysis and Control. Basel: Springer; 2016.

7. Seuret A, Gouaisbaut F. Hierarchy of LMI conditions for the stability analysis of time-delay systems. Syst Control Lett. 2015;81:1-7.
8. Hien LV, Trinh H. Refined Jensen-based inequality approach to stability analysis of time-delay systems. IET Control Theory Appl. 2015;9:2188-2194.
9. Hien LV, Trinh H. New finite-sum inequalities with applications to stability of discrete time-delay systems. Automatica. 2016;71:197-201.
10. Wu J, Lu G, Wo S, Xiao X. Exponential stability and stabilization for nonlinear descriptor systems with discrete and distributed delays. Int J Robust Nonlinear Control. 2013;23:1393-1404.
11. Du NH, Linh VH, Mehrmann V, Thuan DD. Stability and robust stability of linear time-invariant delay differential-algebraic equations. SIAM J Matrix Anal Appl. 2013;34:1631-1654.
12. Hien LV, Vu LH, Phat VN. Improved delay-dependent exponential stability of singular systems with mixed interval time-varying delays. IET Control Theory Appl. 2015;9:1751-8644.
13. Bejarano FJ, Zheng G. Observability of singular time-delay systems with unknown inputs. Syst Control Lett. 2016;89:55-60.
14. Feng Z, Li W, Lam J. Dissipativity analysis for discrete singular systems with time-varying delay. ISA Transactions. 2016;64:86-91.
15. Kaczorek T. Two-Dimensional Linear Systems. Berlin: Springer-Verlag; 1985.
16. Rogers E et al. Multidimensional control systems: case studies in design and evaluation. Multidim Syst Signal Process. 2015;26:895-939.
17. Hien LV, Trinh H. On reachable set estimation of two-dimensional systems described by the Roesser model with time-varying delays. Int J Robust Nonlinear Control. 2018;28:227-246.
18. Hien LV, Trinh H, Lan Huong NT. Delay-dependent energy-to-peak stability of 2-D time-delay Roesser systems with multiplicative stochastic noises. IEEE Trans Autom Control. 2019;64:5066-5073.
19. Hien LV, Trinh H. Prediction-based approach to stabilization of 2-D continuous-time Roesser systems with directional input delays. J Frankl Inst. 2020;357:4779-4794.
20. Hien LV, Trinh H, Pathirana P. On ℓ_1-gain control of 2-D positive Roesser systems with directional delays: Necessary and sufficient conditions. Automatica. 2020;112:1-10.
21. Chen SF. Stability analysis and stabilization of 2-D singular Roesser models. Appl Math Comput. 2015;250:779-91.
22. Xu H, Zou Y. H_∞ control for 2-D singular delayed systems. Int J Syst Sci. 2011;42:609-619.
23. Kririm S, Hmamed A, Tadeo F. Analysis and design of H_∞ controllers for 2D singuar systems with delays. Circuit Syst Signal Process. 2016;35:1579-1592.
24. Hien LV, Vu LH, Trinh H. Stability of two-dimensional descriptor systems with generalized directional delays. Syst Control Lett. 2018;112:42-50.
25. Hien LV. An explicit criterion for finite-time stability of linear nonautonomous systems with delays. Appl Math Lett. 2014;30:12-18.
26. Zhang G, Wang W. Finite-region stability and finite-region boundedness for 2D Roesser models. Math Meth Appl Sci. 2016;39:5757-5769.
27. Zhang G, Trentelman HL, Wang W, Gao J. Input-output finite-region stability and stabilization for discrete 2-D Fornasini-Marchesini models. Syst Control Lett. 2017;99:9-16.
28. Zhang G. Input-output finite-region stability and stabilization for discrete the 2-D Roesser model. J Visual Commun Image Represent. 2018;57:253-261.