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# Imaginary part of microscopic optical potential at negative energies

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**Abstract.** In this paper, we calculate the diagonal contributions $W(R, s=0)$ of the imaginary part of the microscopic optical potential at negative energies, where $W(R,s)=\sum_{lj}\frac{2j+1}{4\pi}\text{Im}\Delta\Sigma_{lj}(r,r',\omega)$, with $R=\frac{1}{2}(r+r')$ corresponds to the radius and shape of $\text{Im}\Delta\Sigma$, and $s=r-r'$. To do it, the microscopic optical potential has been calculated, using the nuclear structure approach, which is based on the Green function method. The coupling between the particle and collective phonon has been performed to calculate the dynamic part of the optical potential. It has been found that the imaginary part at negative energies is very small, as expected. The calculated $W(R,0)$ is maximum on the surface and decreases to zero in the interior.

**Keywords:** Optical potential; Elastic scattering; Microscopic optical model

## 1 Introduction

Optical potential (OP) is one of the most important ingredients for computational codes in nuclear reactions. These potentials (known as the phenomenological optical potential) are usually fitted from scattering data due to the underlying complex many-body problem and unknown nuclear force. However, these potentials do not have the prediction power, for example, for the unstable nuclei region, where experimental data are very scarce. The microscopic optical potentials are expected to be the only existing vehicle to study the exotic nuclei region. In the low-energy region ($E<50$ MeV), the nuclear structure models (NSM) succeeded in building the microscopic optical potential to describe the nucleon elastic scattering [1] by $^{208}$Pb, neutron elastic scattering [2] by $^{16}$O, proton inelastic scattering [3] by $^{24}$O, nucleon elastic scattering by $^{40}$Ca and $^{48}$Ca [4, 6, 7], and nucleon elastic scattering [5, 8] by $^{16}$O, $^{40}$Ca, $^{48}$Ca, and $^{208}$Pb. The nuclear structure approach is based on the Green function method, where the effects of collective states and the Pauli principle are naturally taken into account. This formalism provides a systematic framework to study the optical potential, and it includes automatically the effects of antisymmetry and target recoil. The optical potential is defined as a one-body potential, which can simplify the complex $A+1$ problem. This potential can be represented by the mass operator of the one-particle Green function [9].

## 2 Main text

The optical potential, which describes the motion of a particle in a non-local potential, is

$$V(\vec{r};\vec{r}';E)=\sum(\vec{r};\vec{r}';E)$$
$$=-\int_{-\infty}^{+\infty} e^{iE(t-t')}\sum(\vec{r};\vec{r}';t-t')d(t-t'), \quad (1)$$

where $E$ is the energy of the $(A+1)$ system. This self-energy $\Sigma$ can naturally establish a link between the nuclear structure (negative energy) and nuclear reactions (positive energy). It contains three terms:

$$\Sigma=\Sigma_1+\Sigma_2+\Sigma_3-2\Sigma^{(2)}. \quad (2)$$

To simplify the complex many-body problem inside the mass operator, we suppose [1]:

(i) only the correlations between two nucleons are taken into account;

(ii) the complex function $G_1$ is approximated by the HF mean-field operator $G_{\mathrm{HF}}$;

(iii) the p-p (h) propagator is approximated by the ladder (bubble) approximation, respectively.

These assumptions allow us to reduce the complex many-body problem to a simple one-body potential

$$\Sigma(x,x')=\Sigma_{HF}(x,x')+\Sigma_{RPA}(x,x')-\Sigma'(x,x'). \quad (3)$$

In the case of effective zero-range Skyrme interaction, the optical potential could be simplified more by

$$\Sigma_{opt}=\Sigma_{HF}+\Delta\Sigma(\omega)=\Sigma_{HF}+\Sigma(\omega)-\frac{1}{2}\Sigma^{(2)}(\omega), \quad (4)$$

where $\Sigma_{\mathrm{HF}}$ is the static, real, local, and energy-independent Skyrme Hartree-Fock potential. The dynamic part $\Sigma(\omega)$ is generated from the coupling particle to the collective motion at small amplitudes of the nuclear target. The second-order potential $\Sigma^{(2)}(\omega)$ is taken into account to remove the contribution of the states that violate the Pauli principle in the RPA calculation.

Recently, the elastic scattering observables have been directly linked with the underlying nucleon-nucleon (*NN*) effective phenomenological interactions [1, 2, 4, 5]. These effective interactions (mostly the finite-range Gogny and zero-range Skyrme interactions) have been implemented in the self-consistent mean-field approach and beyond. These approaches are well known since they can provide a good description of both nuclear structure properties as well as nuclear matter around its saturation density. These effective interactions have been successfully used to describe the nucleon elastic scattering for all the double-closed shell nuclei $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{208}$ Pb [1-8, 10, 11]. It is interesting to note that the MOPs at the positive and negative energies are naturally and consistently connected since these potentials are based on the self-energy extracted from the mass operator in the framework of the many-body Green function method.

To calculate the (d,p) reactions, the *NA* optical potential and properties of the bound state of $A+1$ nuclear system. Within the nuclear structure model, the optical potential at negative energies describes the odd-even system, including the odd nucleon and the even-even core. With the commonly used dispersive optical potential, the properties of this $A+1$ system are usually extracted from the extrapolation from the fitting with experimental data in the positive energies (scattering region). In our model, we do not need to use extrapolation since the link between two regions has been naturally established.

Below, we show the conventional zero-range effective Skyrme interaction

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$$V_{skyrme}(\mathbf{r}_1,\mathbf{r}_2) = t_0\left(1+x_0P^\sigma\right)\delta(\mathbf{r})$$
$$+\frac{1}{2}t_1\left(1+x_1P^\sigma\right)\left[\mathbf{k}'\delta(r)+\delta(\mathbf{r})\mathbf{k}^2\right]+t_2\left(1+x_2P^\sigma\right)\mathbf{k}'\delta(\mathbf{r})\mathbf{k}$$
$$+iW_0\left(\vec{\sigma}_1+\vec{\sigma}_2\right).\left[\mathbf{k}'\times\delta(\mathbf{r})\mathbf{k}\right]$$
$$+\frac{1}{6}t_3\left(1+x_3P^\sigma\right)\rho^\alpha(\mathbf{R})\delta(\mathbf{r})$$

$t_0$ is central term

$t_1, t_2$ are velocity-dependent terms,

$W_0$ is spin-orbit term,

$t_3$ is density-dependent term,

(5)

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$, $\mathbf{k}=\frac{1}{2i}(\vec{\nabla}_1-\vec{\nabla}_2)$, $\mathbf{k}'$ is the hermitian conjugate of $\mathbf{k}$ (acting on the left), $P^\sigma=\frac{1}{2}(1+\vec{\sigma}_1.\vec{\sigma}_2)$ is the spin-exchange operator, and $\rho$ is the total density. To obtain the parameters $t_0,t_1,t_2,t_3,W_0,\alpha,x_0,x_1,x_2,x_3$, experimental data such as binding energies and radius have been used in the fitting process. The effective interaction has been fully and consistently used to describe the mean field, the collective states at small amplitudes, and the coupling particle-phonon.

According to Refs. [5, 8, 10, 11], the one-body microscopic optical potential is given as

$$V_{\text{opt}} = V_{\text{HF}} + \Delta\Sigma(\omega), \quad (6)$$

where

$$\Delta\Sigma(\omega) = \Sigma(\omega) - \frac{1}{2}\Sigma^{(2)}(\omega). \quad (7)$$

In Eqs. (6) and (7), $V_{\text{HF}}$ is a static, real, local, and energy-independent Skyrme-Hartree-Fock mean field. The first order, $\Sigma(\omega)$, is the contribution from the particle-vibration coupling calculated as in Refs. [5, 13, 14]. This dynamical potential is non-local, complex, and energy-dependent. The symbol $\omega$ is the nucleon's incident energy. The second-order potential, $\Sigma^{(2)}(\omega)$, is taken into account to treat the issue of the Pauli principle correction. The NN effective phenomenological interaction SLy5 [12] has been adopted. Note that all parameters are fixed and are the same as in Refs. [5, 8, 10, 11].

## 3 Results and discussion

To see the imaginary part of the MOP at negative energies, we consider the diagonal contributions $W(R,s=0)$ of the imaginary part, where $W(R,s)=\sum_{lj}\frac{2j+1}{4\pi}\text{Im}\Delta\Sigma_{lj}(r,r',\omega),$ with $R=\frac{1}{2}(r+r')$ corresponding to the radius and shape of $\text{Im}\Delta\Sigma$, and $s=r-r'$ being its non-locality. Fig. 1 shows the calculations of $W(R,s=0)$ for neutron elastic scattering by $^{208}$Pb at $-2.94$ MeV and $-3.94$ MeV. The negative energy $-3.94$ MeV corresponds to the single particle state $2g9/2$.

The obtained results show that the imaginary part of the optical potential at negative energies is small but different from zero. This coupling to the core is well known as the core polarization, which is described (only the real part) by other nuclear structure models such as the odd-even HF+BCS model [15]. The imaginary part has the strongest effects on the surface and no effect on the interior. Also, by comparing the results at two energies, the imaginary part is larger for the level that is closer to the Fermi level since the coupling to the core is stronger. This work is a further step in our project to explore as much as possible the structure information from the microscopic optical potential.

The obtained results show that the imaginary part of the MOP is different from zero at negative energies. However, the magnitude is very small compared with that at positive energies.

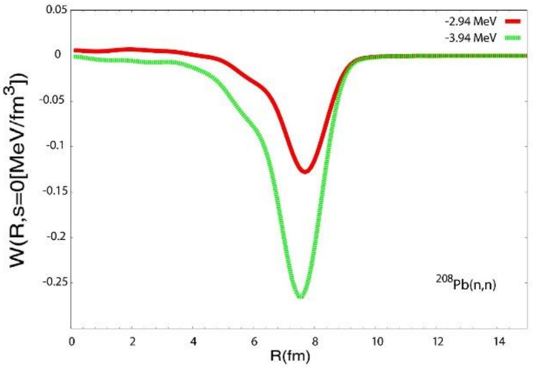


**Fig. 1.** The calculated $W(R, s=0)$ for neutron elastic scattering by $^{208}$Pb with the full effective SLy5 Skyrme interaction. The solid (dashed) curve shows the calculation at -2.94 (-3.94) MeV, respectively

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