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Application of generalized photon-added pair coherent state to quantum teleportation via atom-field entangled channel

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Abstract. In this paper, we introduce a field-atom entangled state that represents the entanglement over time between a two-level atom and a field in a generalized photon-added pair coherent state in the Jaynes-Cummings model. This entangled state is applied for quantum teleportation of an unknown atomic state from a sender to a receiver that is geographically distant. Accordingly, we use an average fidelity criterion to quantify quantum teleportation processing. The results have shown that the process of teleportation is influenced by parameters such as initial field strength, the amplitude of a state to be teleported, and the number of added photons to two modes of the field. In addition, we also compare the success of teleportation in two cases of the field: the pair coherent state and the generalized photon-added pair coherent state.

Keywords: Quantum teleportation, atom-field entangled channel, photon-added states

1 Introduction

The definition of quantum entanglement was first introduced by Schrodinger in 1935 to explain the Einstein-Podolsky-Rosen paradox [1], which pioneered the beginning of quantum information and quantum computer. In these areas, quantum computing and quantum information attracted not only theoretical scientists but also experimental scientists, which has the ability to open a new technological revolution in quantum information in the future [2]. The idea of quantum teleportation was offered by Bennet in 1993 [3] to teleport an unknown quantum state from Alice to Bob at an enormous distance in space. This leads to a number of experimental results showing the correctness and possibility of this idea. Additionally, the process of quantum teleportation is also successful with different distances among photons and atoms, such as the distance between a qubit system with a five-atom system at 600 meters in 2004 [4], and qubit-photon systems over 143 kilometers in 2012 [5]. In particular, quantum teleportation was carried out successfully from light beams to the vibrational states of a diamond on a macroscopic scale in 2016 [6].

A state is called a nonclassical state if it exhibits one or a variety of nonclassical properties, such as the squeezing effect, antibunching, entanglement degree, and sub-Poisson statistics, which have been applied in quantum optics and quantum information [7]. Recently, nonclassical states via adding/subtracting photons to any classical or nonclassical state have the ability to enhance nonclassicality than the initial state such as photon-added squeezing-enhanced coherent state [8], photon-added-and-subtracted two modes pair coherent state [9], two-mode squeezed vacuum states [10], generalized photon-added pair coherent state [11]. A variety of new nonclassical states, such as added/subtracted photon states, with the enhancement of nonclassicality, have the capability to become effective resources in performing quantum tasks. From this, nonclassical states have been proposed, studied, and applied to implementing protocols and tasks in quantum communication [12, 13].

To implement quantum teleportation, Alice and Bob need to share an entangled atom-field channel. The new nonclassical properties mentioned above are seen as entangled resources to teleport the quantum state of photons, particle number state, and pair coherent state in ideal conditions without time independence. However, the teleporting photon state, or any state of the quantum field, always mentions the interaction between states and materials in reality. This leads to a number of protocols that have been proposed to take advantage of quantum teleportation of an atomic state or quantum field state, which is an atom-field-entangled quantum channel [14, 15]. These channels are atom-field entangled states in the Jaynes-Cummings (JC) model that considers the characteristic of entangled dynamics over time in the interaction between atom and field. In this paper, we introduce an effective resource in quantum teleportation, which is an atom-field-entangled state with the field in the generalized photon-added pair coherent state in Section 2. In Section 3, we examine the quantum teleportation of an unknown qubit from Alice to Bob with an atom-field entangled quantum channel. These results will be discussed in detail in Section 4 with figures. Therefore, the main results of this paper will be summarized in the conclusion.

2 Entangled atom-field state

2.1 Generalized photon-added pair coherent state

Considering a boson field with two modes, a and b correspond to two annihilation operators $\hat{a}$ and $\hat{b}$, respectively. The pair coherent state $|\xi, q\rangle$ is defined as the eigenstate of the pair annihilation operator, corresponding to the eigenvalue ξ [16]. In Fock space, the pair coherent state (PCS) has the form

$$|\xi,q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n+q,n\rangle, , \tag{1}$$

in which ξ is a complex number, $\xi = |\xi| e^{i\phi}$ with $|\xi|$ and ϕ are any real numbers, an integer q is the variance of photons between two modes; a normalized constant N_q is determined by $N_q = \left[\sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n!(n+q)!} \right]^{-1/2}$, and $|n+q,n\rangle = |n+q\rangle \otimes |n\rangle$ is a two-mode Fock state.

The generalized photon-added pair coherent state (GPAPCS) is created by acting the photon creation operator on two modes of the pair coherent state [11]. In Fock space, the GPAPCS has the form

$$|\xi,q;m,k\rangle = C_{q;m,k} \sum_{n=0}^{\infty} R_n |n+q+m,n+k\rangle, \tag{2}$$

in which m and k are the number of photons added to two modes a and b of the PCS, and the expansion coefficient has the form

$$C_{q;m,k} = \left[\sum_{n=0}^{\infty} \frac{|\xi|^{2n} (n+q+m)!(n+k)!}{[n!(n+q)!]^2} \right]^{-1/2}, \tag{3}$$

$$R_n = \frac{\xi^n \sqrt{(n+q+m)!(n+k)!}}{n!(n+q)!}. \tag{4}$$

In Eqs. (3) and (4), if we select $m = k$, GPAPCS in Eq. (2) becomes PCS, which corresponds to Eq. (1). Both the pair coherent state

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and the generalized photon-added pair coherent state exhibit nonclassical properties such as the squeezing effect, antibunching, sub-Poisson statistics, and entanglement degree [11, 16]. These are effectively nonclassical properties for quantum tasks.

2.2 The Hamiltonian of the atom-field system in the two-mode Jaynes-Cummings model

Examining atom-field interaction in the two-mode JC model includes an effective two-level atom that interacts with the two-mode field in the generalized photon-added pair coherent state. An atom has two levels: excited level $|e\rangle$ and ground level $|g\rangle$, in which a movement between two modes is taken by two-photon processes. In these levels, an atom also has intermediary level $|i\rangle$, in which communication between two levels ($|g\rangle$ and $|i\rangle$) and two levels ($|e\rangle$ and $|i\rangle$) is allowed, while direct communication between two levels($|e\rangle$ and $|g\rangle$) is forbidden. In rotating-wave approximation, the Hamiltonian system was examined without the Stark effect, which has the form [17]:

$$\hat{H}=\omega\hat{S}_z+\omega_1\hat{a}^+\hat{a}+\omega_2\hat{b}^+\hat{b}+\lambda\left(\hat{a}^+\hat{b}^+|g\rangle\langle e|+\hat{a}\hat{b}|e\rangle\langle g|\right), \quad (5)$$

in which ω_1 and ω_2 correspond to mode a and mode b of the field, $\omega=\omega_e-\omega_g$ is the frequency of the atom that satisfy the condition $\omega_e-\omega_i=\omega_1+\delta, \omega_i-\omega_g=\omega_2-\delta$ with δ is the modulation of the one-photon process, λ is the interaction constant between the atom and the field, and $\hat{S}_z=\frac{1}{2}(|e\rangle\langle e|-|g\rangle\langle g|)$ is the displacement operator of the atom. In this paper, we consider units of system $\hbar=c=1$.

To be simple, we choose two basic vectors, $|e,n+q+m,n+k\rangle$, and $|g,n+q+m+1,n+k+1\rangle$; these vectors are called dressed states. Eigenstates of the Hamiltonian in Eq. (5) are written in dressed states as follows:

$$\left|\kappa_n^{\pm}\right\rangle=\frac{1}{\sqrt{2}}\left(|e,n+q+m,n+k\rangle\pm|g,n+q+m+1,n+k+1\rangle\right). \quad (6)$$

Solving the eigenstates of the Hamiltonian $\hat{H}$ in Eq. (5), we get

$$\lambda_n^{\pm}=\omega_1(n+q+m)+\omega_2(n+k)+\frac{1}{2}\omega\pm\beta_n, \quad (7)$$

in which the constant β_n is determined by

$$\beta_n=\sqrt{(n+q+m)(n+k)}. \quad (8)$$

2.3 Atom-field entangled state

Considering a system includes an atom and field when not interacting with each other at the initial time, which exhibits via a state as

$$|\psi(0)\rangle=|A\rangle\otimes|F\rangle, \quad (9)$$

in which A and F stand for the atom and the field, respectively. At the initial time, the atom is in the excited state, and the field is in the GPAPCS that is determined in Eq. (2). From this, Eq. (9) can be written as

$$|\psi(0)\rangle=C_{q;m,k}\sum_{n=0}^{\infty}R_n|e,n+q+m,n+k\rangle, \quad (10)$$

with coefficients $C_{q;m,k}$ and R_n are determined in Eqs. (3) and Eq. (4).

At anytime $t>0$, the wave function of the system $|\psi(t)\rangle$ in interactive pictures has the form

$$|\psi(t)\rangle=\hat{U}(t)|\psi(0)\rangle, \quad (11)$$

where $\hat{U}(t)$ is a time-evolving operator. In a basic system of dressed states, we can expand the unitary operator as

$$\hat{U}(t)=U_{ee}(n,t)|e\rangle\langle e|+U_{eg}(n,t)|e\rangle \times\langle g|+U_{ge}(n,t)|g\rangle\langle e|+U_{gg}(n,t)|g\rangle\langle g|, \quad (12)$$

in which matrix elements $U_{ij}(n,t)$ with indexes $i,j=\{e,g\}$ are determined by the Hamiltonian in Eq. (5), and eigenstates and eigenvalues correspond to Eqs. (6) and (7), which have the form as follows:

$$\begin{aligned} U_{ee}(n,t)&=U_{gg}(n,t)=\frac{1}{2}\left(e^{-i\lambda_n^{+}t}+e^{-i\lambda_n^{-}t}\right),\\ U_{eg}(n,t)&=U_{ge}^{*}(n,t)=\frac{1}{2}\left(e^{-i\lambda_n^{+}t}-e^{-i\lambda_n^{-}t}\right). \end{aligned} \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11), we get an explicit formula for the state that describes the evolution over time

$$\begin{aligned} |\psi(t)\rangle=C_{q;m,k}\sum_{n=0}^{\infty}R_n[&\cos(\lambda\beta_n t)|e,n+q+m,n+k\rangle\\ &-i\sin(\lambda\beta_n t)|g,n+q+m+1,n+k+1\rangle] \end{aligned} \tag{14}$$

in which coefficient β_n is in Eq. (8). The state of the system over time in Eq. (14) is the atom-field entangled state, in which the entanglement degree between an effective two-level atom and two-mode field of the GPAPCS that circulate over time. Research shows that the more photons are added to two modes of the field of the GPAPCS with the same values of photons, the greater the degree of entanglement between atom and field [17]. The enhancement of entanglement degree for entanglement resources plays a vital role in carrying out quantum tasks such as quantum teleportation [12, 13].

3 Quantum teleportation via atom-field entangled quantum channel

Quantum teleportation is a method to teleport an unknown state from Alice to Bob at any distance in space via an entangled state shared by a combination of Alice and Bob with a classical channel. In the process of teleportation, all manipulation is taken by Alice and Bob on their states without any security [2]. All information is teleported with high accuracy and maximum security, which is the idea of quantum teleportation. The idea of quantum teleportation was first introduced by Bennet and partners in 1993 [3]. Studies have proposed entangled resources for this process later. A variety of studies were examined by combining Fock state or pair coherent state with entangled resources such as two-mode photon-added displaced squeezed states [7], photon-added-and-subtracted two modes pair coherent state [9], two-mode squeezed vacuum states [10], and other resources. Besides, a number of other protocols were proposed to teleport not only normal states but also atomic states [13, 14, 18]. Using the atom-field system in the JC model plays an important role in making useful resources and achieving desired results. In this section, we use the atom-field entangled channel in Eq. (14) to implement quantum teleportation of an unknown atom state from Alice to Bob.

Using Eq. (14), the density matrix operator over time, $\hat{\rho}_{af}(t)$, has the form

$$\begin{aligned} \hat{\rho}_{af}(t)&=|\psi(t)\rangle\langle\psi(t)|\\ &=\sum_{n=0}^{\infty}\Big[C_{1,n}(t)|e,n+q+m,n+k\rangle\langle e,n+q+m,n+k|\\ &+C_{2,n}(t)\big[|e,n+q+m,n+k\rangle\langle g,n+q+m+1,n+k+1|\\ &-|g,n+q+m+1,n+k+1\rangle\langle e,n+q+m,n+k|\big]\\ &+C_{3,n}(t)|g,n+q+m+1,n+k+1\rangle\langle g,n+q+m+1,n+k+1|, \end{aligned} \tag{15}$$

in which time-dependent constants $C_{i,n}(t)$ are determined by

$$\begin{aligned} C_{1,n}(t)&=|C_{q;m,k}|^2|R_n|^2\cos^2(\lambda\beta_n t),\\ C_{2,n}(t)&=-i|C_{q;m,k}|^2|R_n|^2\sin(\lambda\beta_n t)\cos(\lambda\beta_n t),\\ C_{3,n}(t)&=|C_{q;m,k}|^2|R_n|^2\sin^2(\lambda\beta_n t). \end{aligned} \tag{16}$$

The density matrix operator of the system over time $\hat{\rho}_{af}(t)$ in Eq. (15) is indicated in $(2\times\infty)$ space dimensions. To implement quantum teleportation of a qubit from sender to receiver, we are only interested in finite dimensions (2×2), in which an atom corresponds to two levels: excited level and ground level, and also the field corresponds to two states: $|n+q+m,n+k\rangle$ and $|n+q+m+1,n+k+1\rangle$ [19]. Therefore, the operator $\hat{\rho}_{af}(t)$

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can be written in (2×2) space dimensions in detail as

$$\begin{aligned}\hat{\rho}_{af}(t)=&C_{1,n}(t)|e,n+q+m,n+k\rangle\langle e,n+q+m,n+k|\\&+C_{1,n+1}(t)|e,n+q+m+1,n+k+1\rangle\langle e,n+q+m+1,n+k+1|\\&+C_{2,n}(t)\big[|e,n+q+m,n+k\rangle\langle g,n+q+m+1,n+k+1|\\&-|g,n+q+m+1,n+k+1\rangle\langle e,n+q+m,n+k|\big]\\&+C_{3,n-1}(t)|g,n+q+m,n+k\rangle\langle g,n+q+m,n+k|\\&+C_{3,n}(t)|g,n+q+m+1,n+k+1\rangle\langle g,n+q+m+1,n+k+1|\end{aligned} \tag{17}$$

with the coefficients $C_{i,n}(t)$ are determined in Eq. (16).

Eq. (17) can be written in computer language as follow:

$$\begin{aligned}\hat{\rho}_{af}(t)=&C_{1,n}(t)|10\rangle\langle 10|+C_{1,n+1}(t)|11\rangle\langle 11|\\&+C_{2,n}(t)\big(|10\rangle\langle 01|-|01\rangle\langle 10|\big)\\&+C_{3,n-1}(t)|00\rangle\langle 00|+C_{3,n}(t)|01\rangle\langle 01|,\end{aligned} \tag{18}$$

in which the states $|g, n+q+m, n+k\rangle$ and $|e, n+q+m+1, n+k+1\rangle$ can be written as $|00\rangle$ and $|11\rangle$, and the states $|g, n+q+m+1, n+k+1\rangle$ and $|e, n+q+m, n+k\rangle$ can be written as $|01\rangle$ and $|10\rangle$, respectively. Instead of exhibiting resources by the atom-field entangled state in Eq. (14), we can use Eq. (18) to replace Eq. (14) in calculating quantum teleportation. The operator $\hat{\rho}_{af}(t)$ in Eq. (18) is a mixed state of two qubits that can be transported into Bell-diagonal states [2, 19] and has the form

$$\begin{aligned}|\phi^{\pm}\rangle&=\frac{1}{\sqrt{2}}\big(|00\rangle\pm|11\rangle\big),\\|\chi^{\pm}\rangle&=\frac{1}{\sqrt{2}}\big(|01\rangle\pm|10\rangle\big).\end{aligned} \tag{19}$$

Assuming that Alice needs to teleport an unknown atomic state, which is described in the form

$$\hat{\rho}_u=|\mu|^2|0\rangle\langle 0|+\mu\nu^*|0\rangle\langle 1|+\mu^*\nu|1\rangle\langle 0|+|\nu|^2|1\rangle\langle 1|, \tag{20}$$

in which states $|0\rangle$ and $|1\rangle$ replaced states $|e\rangle$ and $|g\rangle$ of the atom, coefficients μ and ν are the amplitudes of the entangled state that satisfy the condition $|\mu|^2+|\upsilon|^2=1$. This leads to the system including Alice containing qubit 1 in Eq. (20) and qubit 2 being in the field of GPAPCS; also, qubit 3 is in Bob with the two-level atomic state.

To carry out the quantum teleportation of qubit 1, Alice and Bob need to share an entangled state, which is an atom-field entangled channel in Eq. (18). Later, Alice realizes a measurement of qubit 1 and qubit 2 by Bell state and sends results to Bob by classical channel. From Alice's results, Bob continuously realizes a unitary transformation to qubit 3 to get the state of qubit 1 [2]. This teleportation needs to be estimated by the average fidelity.

At the initial time, a state of three-qubit systems is described by

$$\hat{\rho}_{u,af}=\hat{\rho}_u\otimes\hat{\rho}_{af}(t). \tag{21}$$

If Alice implements a measurement on one of the four Bell states $|\chi^+\rangle\langle\chi^+|$, Bob will receive results as

$$\hat{\rho}_B=\varepsilon_1|0\rangle\langle 0|+\varepsilon_2|0\rangle\langle 1|-\varepsilon_3|1\rangle\langle 0|+\varepsilon_4|1\rangle\langle 1|, \tag{22}$$

in which coefficients ε_i are determined by

$$\begin{aligned}\varepsilon_1&=\frac{|\mu|^2C_{1,n}(t)+|\upsilon|^2C_{3,n-1}(t)}{2N},\quad \varepsilon_2=\frac{\mu\upsilon^*C_{2,n}(t)}{2N},\\\varepsilon_3&=\frac{\mu^*\upsilon C^*_{2,n}(t)}{2N},\quad \varepsilon_4=\frac{|\mu|^2C_{1,n+1}(t)+|\upsilon|^2C_{3,n}(t)}{2N},\end{aligned} \tag{23}$$

with N is a normalized constant has the form

$$N=\frac{1}{2}\Big\{|\mu|^2\big[C_{1,n}(t)+C_{1,n+1}(t)\big]+|\upsilon|^2\big[C_{3,n-1}(t)+C_{3,n}(t)\big]\Big\}. \tag{24}$$

The success of the process of teleportation is estimated by the average fidelity F_{av}. If the value of average fidelity F_{av} is equal to 1, this leads to the output state $\hat{\rho}_B$ being the same as the input state $\hat{\rho}_u$. Additionally, the process of teleportation is unsuccessful, while the average fidelity F_{av} is equal to 0. Using Eqs. (20) and (22), we get

$$F_{av}=\mathrm{Tr}\big(\hat{\rho}_B\hat{\rho}_u\big)=|\mu|^2\varepsilon_1+|\upsilon|^2\varepsilon_4, \tag{25}$$

in which μ and ν are the amplitudes of the entangled state, and coefficients ε_i are determined in Eq. (23). The results in Eq. (25) will be discussed in detail in the next section.

4 Results and discussion

Eq. (25) indicates the average fidelity F_{av} as a function of λt, the amplitude of entangled state $|\mu|$, the initial field intensity $|\xi|$, and the photon addition of two modes a and b of the field in the PCS and the GPAPCS. In both cases of the PCS and the GPAPCS, we observe the cyclic oscillation over time, which is similar to the interaction over time between an atom and the field that was mentioned above in an atom-field entangled quantum channel.

In the case of the PCS with fixed parameters $|\xi| = 1$ and $n = 2$ as indicated the field, Fig. 1 shows the average fidelity of F_{av} as a function of λt. We observe that if the amplitude of the entangled state $|\mu|$ increases, the value of F_{av} simultaneously decreases. Besides, the value of $|\mu|$ corresponds to $(0.8, 0.5, 0.3)$, and the maximum value of F_{av} is approximately $(0.6, 0.75, 0.9)$. This means that the smaller the amplitude of the entangled state , the more successful the teleportation process becomes.

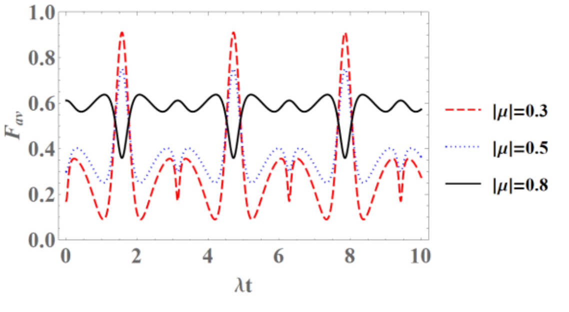

Fig. 1. The dependence of F_{av} on λt with fixed parameters $|\xi| = 1, n = 2$, and the PCS field with $q = m = k = 0$

Fig. 2 indicates the average fidelity of F_{av} as a function of μ with fixed parameters $t = 3/2$ and $n = 2$ in case the PCS is also considered as the field. We observe that the values of F_{av} are divided into two ranges, which correspond to the first range with $0 < \mu < 0.7$ and the second range with $0.7 < \mu < 1$. The values of F_{av} in the first range are proportional to the amplitude $|\xi|$ of the initial field intensity, which leads to the values of F_{av} increasing if the values of $|\xi|$ increase. Otherwise, the more the values of $|\xi|$ increase, the more the values of F_{av} decrease in the second range. From this, we estimate that the process of teleportation only exists when the amplitude of the entangled state lies in the first range. Therefore, if the initial field intensity is greater, the teleportation is more successful.

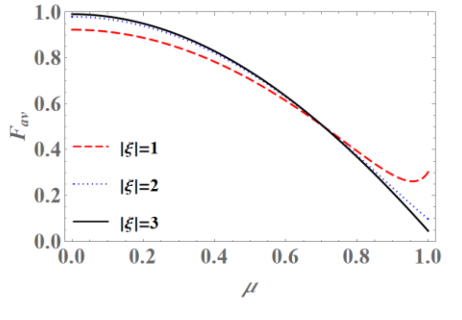

Fig. 2. The dependence of F_{av} on μ with fixed parameters $n = 2, t = 3/2$, and the PCS field with $q = m = k = 0$

Fig. 3 indicates the average fidelity of F_{av} as a function of λt with fixed parameter parameters $|\xi| = 1, n = 2$, and $|\mu| = 0.5$ in the cases where the fields are the PCS (dashed line) and the GPAPCS (solid line). The symbols (0,0) and (2,2) show the parameter set (m, k), which is the photon addition of two modes of the field. In Fig. 3, we observe that by adding photons to two modes, a and b, at the same time, the maximum values of F_{av} in both cases of the PCS and GPAPCS are unchanged. However, the local maximum points in a period and the frequencies in a period of the GPAPCS increase. This implies that the values of the average fidelity F_{av} in

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GPAPCS are higher than the PCS many times in a period. Therefore, the process of quantum teleportation in the GPAPCS is more successful than in the PCS.

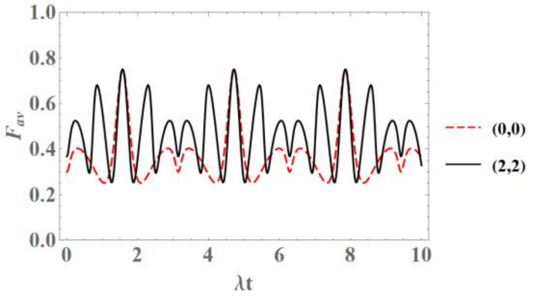

Fig. 3. The dependence of F_{av} on λt for the GPAPCS field with fixed parameters $|\xi| = 1$, $n = 2$, and $|\mu| = 0.5$

To increase the values of F_{av}, we increase the photon addition to two modes of the field and the initial field intensity. Fig. 4 shows the average fidelity of F_{av} as a function of λt with fixed parameters $|\xi| = 2$, $n = 2$, and $|\mu| = 0.3$ in cases where the field is the GPAPCS. Clearly, we observe that if the photon addition to two modes and the initial field intensity increase, frequencies in a period of F_{av} increase. Besides, local maximum points of F_{av} increase many times in a period. This implies that adding photons to modes of the field and changing the parameters of the field intensity play a vital role in implementing quantum teleportation.

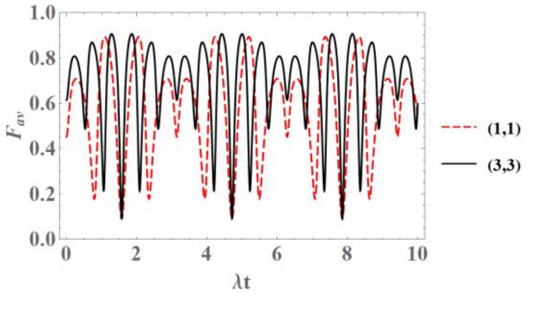

Fig. 4. The dependence of F_{av} on λt for the GPAPCS field with fixed parameters $|\xi| = 2$, $n = 2$, and $|\mu| = 0.3$

5 Conclusion

In this paper, we introduced an atom-field entangled state in the Jayness-Cummings model, in which the field is the generalized photon-added pair coherent state with the enhancement of nonclassical properties. We indicate that the combination of the entangled state with the atom-field entanglement makes a significant and outstanding contribution to quantum teleportation. Besides, we also examine the process of teleportation of an unknown atomic state from Alice to Bob with the resource, which is the atom-field entangled state. The results indicate that the average fidelity has cyclic oscillation over time in the PCS and GPAPCS, and depends on the amplitude of the entangled state, the initial field intensity, and the photon addition of the field. Additionally, the smaller the amplitude of the entangled state, the more successful the process of teleportation. Therefore, using the GPAPCS as the entangled resource and increasing the initial field intensity achieves a better result in quantum teleportation than using the PCS.

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