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TOWARD REWRITING FUZZY GRAPHS BASED ON HEDGE ALGEBRA

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Abstract. In this paper, we study fuzzy graph propeties with combinatorial matrix theory in fuzzy linguistic matrix. We use hedge algebra and linguistic variables for rewriting and reasoning with words. We figure out theorem of limiting in matix space. We also discover limit space states of fuzzy graph with a nice theorem. This is the important theorem to decide whether automata is finite automata or not.

Keywords: fuzzy logic, linguistic variable, hedge algebra, fuzzy rewriting system

1 Introduction

In everyday life, people use natural language (NL) for analysing, reasoning, and finally to make their decisions. Computing with words (CWW) [5] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy logic (FL) is fuzzy cognitive map (**FCM**), introduced by B. Kosko [1], combined fuzzy logic with neural network. **FCM** has a lots of applications in both modeling and reasoning fuzzy knowledge [3,4] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain [5], for example, linguistic summarization problems [6]. To solve this problem, in the paper, we use an abtract algebra, called hedge algebra (HA) as a tool for computing with words.

The remainder of paper is organized as follows. Section 2 reviews some main concepts of computing with words based on **HA** in subsection 2.1 and describes several primary concepts for

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FCM in subsection 2.2. Section 3 reviews modeling with words using **HA**. Important section 4 proves two theorem, the center of paper. Section 5 outlines discussion and future work.

2 Preliminaries

This section presents basic concepts of **HA** and **FCM** used in the paper.

2.1 Hedge algebra

In this section, we review some **HA** knowledges related to our research paper and give basic definitions. First definition of a **HA** is specified by 3-Tuple **HA** $= (X, H, \leq)$ in [7]. In [8] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so **HA** $= (X, G, C, H, \leq)$ where $H \neq \varnothing, G = \{c^+, c^-\}, C = \{0, W, 1\}$.

Domain of X is $\mathbf{L} = Dom(X) = \{\delta c \mid c \in G, \delta \in H^* (hedge\ string\ over\ H)\}$, $\{\mathbf{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \ldots h_1 c$ is said to be a canonical string of linguistic variable x .

Example 1. Fuzzy subset *X is Age*, $G = \{c^+ = young; c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable Age X is (X) or for short:

L = *{very less young ; less young ; young ; more young ; very young ; very very young ...}*

Fuzziness properties of elements in **HA**, specified by *fm* (fuzziness measure) [8] as follows:
Difinition 2.1. A mapping $:\rightarrow [0,1]$ is said to be the fuzziness measure of **L** if:

1. $\sum_{c \in \{c^+, c^-\}} (c) = 1, (0) = (w) = (1) = 0$.
2. $\sum_{h_i \in H} (h_i x) = (x)$, $x = h_n h_{n-1} \ldots h_1 c$, the canonical form.
3. $(h_n h_{n-1} \ldots h_1 c) = \prod_{i=1}^{n} (h_i) \times \mu(x)$.

2.2 Fuzzy cognitive map

Fuzzy cognitive map (**FCM**), so-called fuzzy graph, is feedback dynamical system for modeling fuzzy causal knowledge, introduced by B. Kosko [1]. **FCM** is a set of nodes, which present concepts and a set of directed edges to link nodes. The edges represent the causal links between these concepts. Mathematically, a **FCM** bis defined by.

Definition 2.2. A **FCM** is a 4- Tuple:

$$\mathbf{FCM} = \{C, E, , f\} \tag{1}$$

In which:

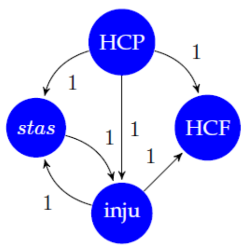
1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.

2. $E : (C_i, C_j) \to e_{ij} \in \{-1, 0, 1\}$ is a function associating e_{ij} with a pair of concepts (C_i, C_j), so that $e_{ij} =$ "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N\times N}$

3. The map: : $C_i \to C_i(t) \in [0,1], t \in N$

4. With $C(0) = [C_1(0, C_2(0), \ldots, C_n(0)] \in [0,1]^N$ is the initial vector, recurring transformation function f defined as:

$$C_j(t+1) = f(\sum_{i=1}^{N} e_{ij} C_i(t)) \tag{2}$$



Example 2. Fig.1 shows a medical problem from expert domain of strokes and blood clotting involving. Concepts C={blood stasis (stas), endothelial injury (inju), hypercoagulation factors (HCP and HCF)} [2]. The conection matrix is::

$$E = (e_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Fig. 1. A simple FCM

FCMs have played a vital role in the applications of scientific areas, including expert system, robotics, medicine, education, information technology, prediction, etc [3,4].

3 Modeling with words

Fuzzy model, based on linguistic variables, is constructed from linguistic hedge of **HA** [10, 11].

Definition 3.1. [Linguistic lattice] With **L** as in the section 2.1, set $\{\wedge, \vee\}$ are logical operators, defined in [7,8], a linguistic lattice $\mathcal{L}$ is a tuple:

$$\mathcal{L} = (, \vee, \wedge, 0, 1) \tag{3}$$

Property 3.1. The following are some properties for $\mathcal{L}$:

1. $\mathcal{L}$ is a linguistic-bounded lattice.

2. $(, \vee)$ and $(, \wedge)$ are semigroups.

Definition 3.2. A linguistic cognitive map (**LCM**) is a 4- Tuple:

$$\textbf{LCM} = \{C, E, , f\} \tag{4}$$

In which:

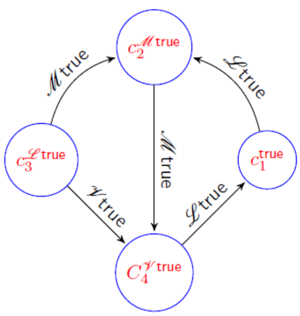
1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.

2. $E : (C_i, C_j) \to e_{ij} \in;\ e_{ij} =$ "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N \times N} \in^{N \times N}$

3. The map: $: C_i \to C_i^t \in, t \in N$

4. With $(0) = [C_1^0, C_2^0, \ldots, C_n^0] \in^N$ is the initial vector, recurring transformation function f defined as:

$$C_j^{t+1} = f\left(\sum_{i=1}^{N} e_{ij} C_i^t\right) \in \tag{5}$$



Example 3. Fig. 2 shows a simple LCM. Let

$$\text{HA} = <\text{X} = \text{truth}; c^{+} = \text{true}; \text{H} = \{L, M, V\}> \tag{6}$$

be a HA with order as $L < M < V$ (L for less, M for more and V for very are hedges).

$C = \{c1, c2, c3, c4\}$ is the set of 4 concepts with corresponding values

$\text{C} = \{\text{true}, M\,\text{true}, L\,\text{true}, V\,\text{true}\}$

Fig. 2. A simple LCM

Square matrix:

$$M = (m_{ij} \in \mathbb{L})_{4 \times 4} = \begin{vmatrix} 0 & \mathscr{L}\text{true} & 0 & 0 \\ 0 & 0 & 0 & \mathscr{M}\text{true} \\ 0 & \mathscr{M}\text{true} & 0 & \mathscr{V}\text{true} \\ \mathscr{L}\text{true} & 0 & 0 & 0 \end{vmatrix}.$$

is the adjacency matrix of **LCM**. Causal relation between c_i and c_j is m_{ij}, for example if $i = 1, j = 2$ then causal relation between c_1 and c_2 is: " *if* c_1 *is* *then* c_2 *is* ***M*** true *is* **L** true" or let ***P***="if c_1 is *true* then c_2 is ***M*** true" be a fuzzy proposition *FP* [9] then $(P) = L\,true$

Definition 3.3. A **LCM** is called complete if between any two nodes always having a connected edge (without looping edges).

4 Rewriting LCM

In many learning algorithms, which use Hebbian rule [3, 4], as time t, the weight of every edges will always be updated $\Delta e_{ij} = f(\sum_k e_{jk} \times c_k \times c_j)$. Let ${}^{\hbar}M(n), 2 \le n \le$ be total connection matrices with vetices. Fuzzifying edge set uses $\hbar$ hedges. The following theorem figures out side of the connection matrix.

Theorem 4.1. Linguistic connection matrix $M = (m_{ij} \in)_{\times}$ constructing on $\hbar$ hedges has size:

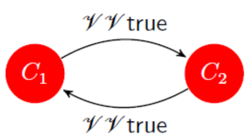
$$ {}^{\hbar}M(N) = (\hbar^{\hbar})^{2\times\binom{N}{2}} \tag{7} $$

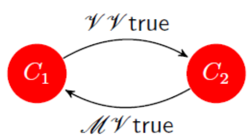
We proof theorem (4.1) by using induction method on number of vetices n, $2 \le n \le$. This process follows two steps. First, set $n = 2$ and check to see if the ${}^{\hbar}M(2)$ is . Next, assume ${}^{\hbar}M(n)$ is , we have to prove ${}^{\hbar}M(n+1)$ is as in logical expression:

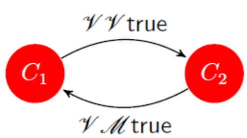
$$ {}^{\hbar}M(2) \wedge ({}^{\hbar}M(n) \to^{\hbar} M(n+1)) \to \forall n\, {}^{\hbar}M(n) \tag{8} $$

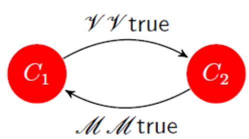
Proof. Without loss of generality, we set $\hbar = 2$ and induction on n. with $\hbar \ge 3$, the process is the same.

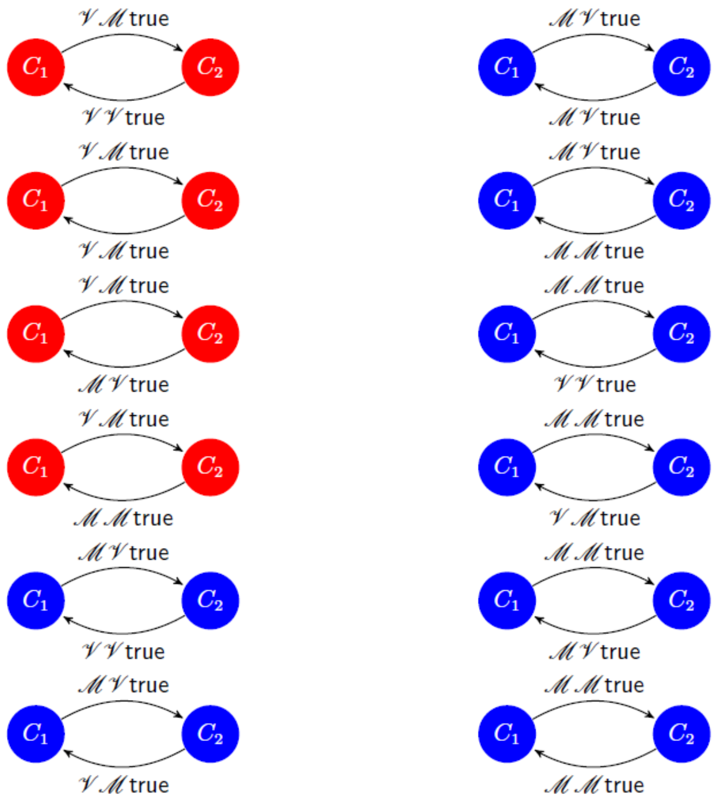
1. To prove ${}^{2}M(2)$ is , we must indicate that: ${}^{2}M(2) = (2^2)^{2\times\binom{2}{2}} = 16$. This is done as in the following 16 figures, in which heges $\{= very, = more\}$











2. Now, assume ${}^2M(n)$ is true on n vetices $c_1, c_2, \ldots, c_n$, that is ${}^2M(n) = (2^2)^{2\times\binom{n}{2}}$. We must prove ${}^2M(n+1) = (2^2)^{2\times\binom{n+1}{2}}$ is true.

consider on vetex c_{n+1}, there have:

- n edges from n vetices $c_1, c_2, \ldots, c_n$ go in c_{n+1}

- n edges from c_{n+1} go out to n vetices $c_1, c_2, \ldots, c_n$

- Total $2 \times n$ edges connected to c_{n+1} which generates $(2^2)^{2\times n}$ difference combinatories.

Applying product rule:

$$
\begin{aligned}
{}^2M(n+1) &= {}^2M(n) \times (2^2)^{2\times n} \\
&= (2^2)^{2\times\binom{n}{2}} \times (2^2)^{2\times n} \\
&= (2^2)^{2\times\binom{n+1}{2}} \\
&\mathcal{QED}.
\end{aligned}
$$

By using the counting method, it is straightforward to prove theorem (4.1) in the case of complete LCM

Theorem 4.1 is important in counting the connection matries. On the other hand, let ${}^{\hbar}LCM(n)$ be total LCM which generate from N veties. We want to know whether ${}^{\hbar}LCM(n)$ finite or infinite. Finding ${}^{\hbar}LCM(n)$ helps to limit searching space in many cases.

Theorem 4.1. Fuzzifying ${}^{\hbar}LCM(n)$ vertices and $2\times\begin{pmatrix}N\\2\end{pmatrix}$ edges use $\hbar$ hedges generated a state space ${}^{\hbar}LCM(n)$:

$$^{\hbar}\mathbb{LCM}(\mathbb{N}) = (\hbar^{\hbar})^{\mathbb{N}^2} \tag{9}$$

Proof. It is straightforward to prove theorem 2(`)@ by using combinatory algebra.

- vetices with $\hbar^{\hbar}$ cases for each vetex which produce $(\hbar^{\hbar})$
- Applying result from theorem 1:

$$\begin{aligned} ^{\hbar}\mathbb{LCM}(\mathbb{N}) &= {}^{\hbar}M(\mathbb{N}) \times (\hbar^{\hbar})^{\mathbb{N}} \\ &= (\hbar^{\hbar})^{2\times\binom{\mathbb{N}}{2}} \times (\hbar^{\hbar})^{\mathbb{N}} \\ &= (\hbar^{\hbar})^{\mathbb{N}^2} \\ &\mathcal{QED}. \end{aligned}$$

5 Conclusions and future work

We have proved two impotant theorems in rewriting fuzzy graphs. First theorem verified connection matrix is limited by expression $(\hbar^{\hbar})^{2\times\begin{pmatrix}N\\2\end{pmatrix}}$. We also demonstrated the theorem about the whole state space is $(\hbar^{\hbar})^{N^2}$. This is the important theorem to indicate that graph state space is finite and therefore automata is finite.

Our next study is as follow:

Give $= \{X, , \{c^+, c^-\}, \{0, W, 1\}, \leq\}$ and let

$$\mathrm{H}^* = \{h_n h_{n-1} \ldots h_0\ c^+ \mid \forall h_i \in ; i \geq 0\} \tag{10}$$

be an alphabet of node and edge labels. A graph over H is a tuple:

$\mathrm{LCM} = \langle V, edg, lab \rangle$ (2)

In which V_{LCM} is the finite set of $vertices$; Binary relation $edg_{LCM} \subseteq V_{LCM} \times H \times V_{LCM}$ saying if two vertices are linked by an edge with label in H. Total map $lab_{LCM} : V_{LCM} \rightarrow H$ assigning a label in H to each vertex of LCM.

The set of all LCM over H is denote LCM_H, and the set of all graphs isomorphic to LCM is denote $[LCM_H]$. A graph language L is a subset $L \subset [LCM_H]$.

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